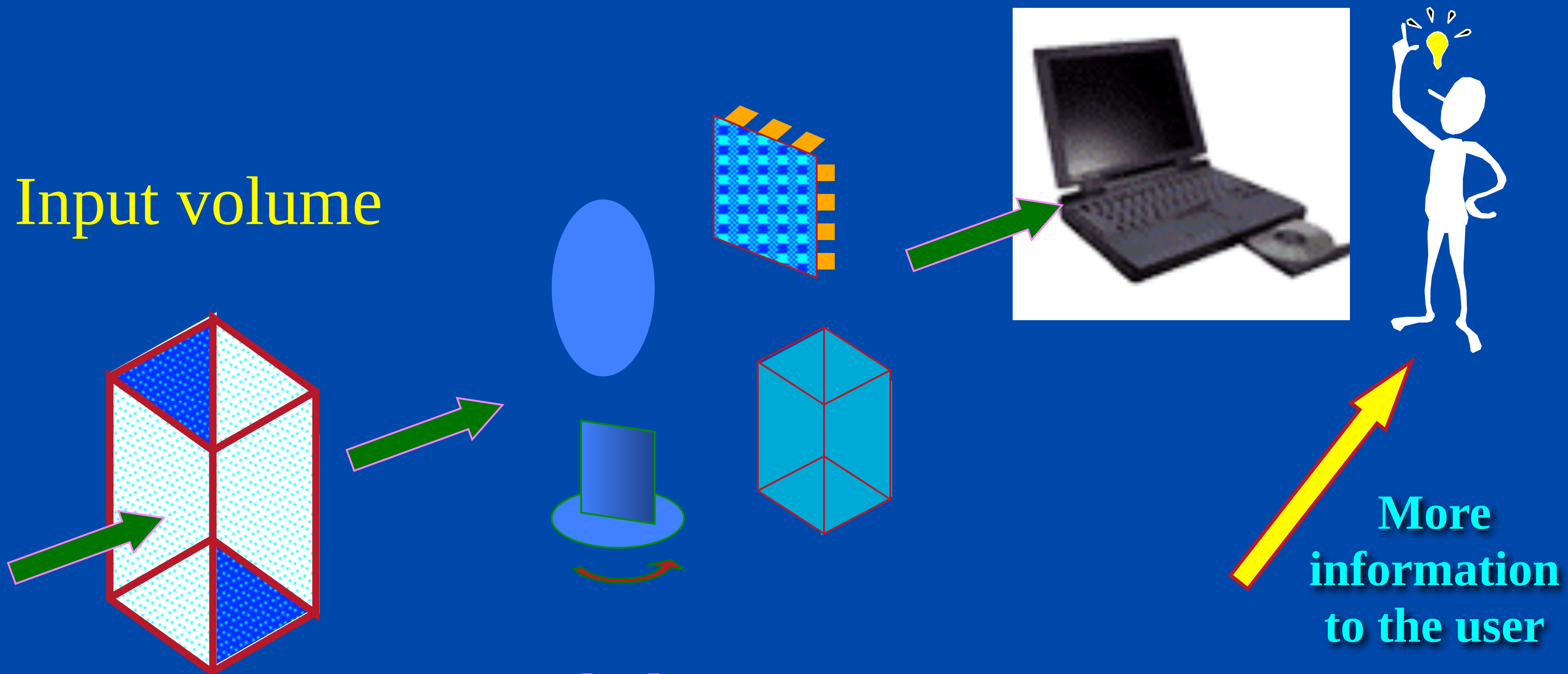
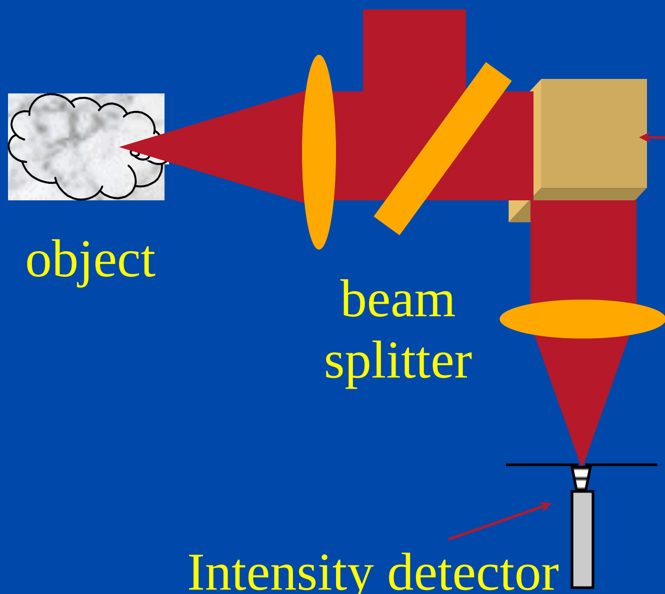
(3+)D Optical Engineering



Optical elements, micro-actuators, smart pixels + Digital processing

in Dave Brady's group, circa ~98-99... co-consipators: Bob Plemmons, Sudhakar Prasad, the late Dennis Healy ...

Confocal microscope with volume holographic filter

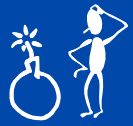


The volume hologram acts as a depth-selective filter through the *Bragg pinhole* effect.

Barbastathis, Balberg, and Brady *Opt. Let*. **24** (12) 811-813, 1999.

Matched filtering is better suited to propagation properties of light

3D scanning is still required to acquire the entire object



Hologram does not diffract 100% of the light ⇒ potential photon collection deficiency

in Dave Brady’s group, circa ~98-99... co-consipators: Bob Plemmons, Sudhakar Prasad, the late Dennis Healy ...

Compressive phase retrieval

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Zhengyun Zhang,[3] Shakil Rehman,[3] Chen Zhi,[3] Justin W. Lee,[4] Adam Pan[4]

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[5]University of California, Berkeley

[6]State University of New York, Albany

Today's talk is about

- Compressive measurements (sparsity priors)
- Coherent light
  - Digital holography and particle localization
- Partially coherent light
  - Phase space and mutual intensity retrieval

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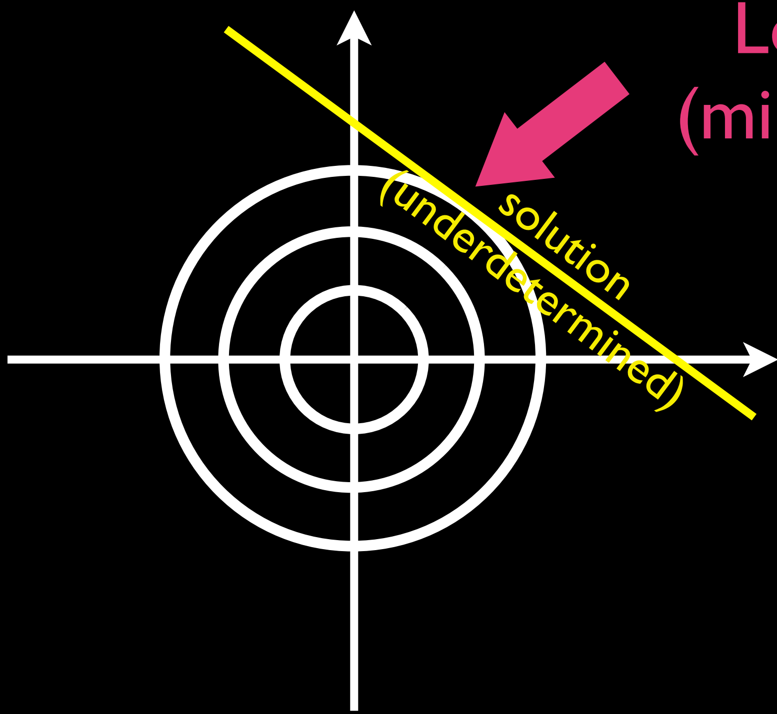
Compressive sensing: a simple-minded example

- You drive by a farm with chicken and sheep and count a total of 8 legs:
  - how many chicken and sheep are there?
- underdetermined - *need another equation like "the total number of heads I count is..."*
- alternatively we can use a "sparsity prior" in the total # of types of animals.
  - *either* chicken *or* sheep

L2



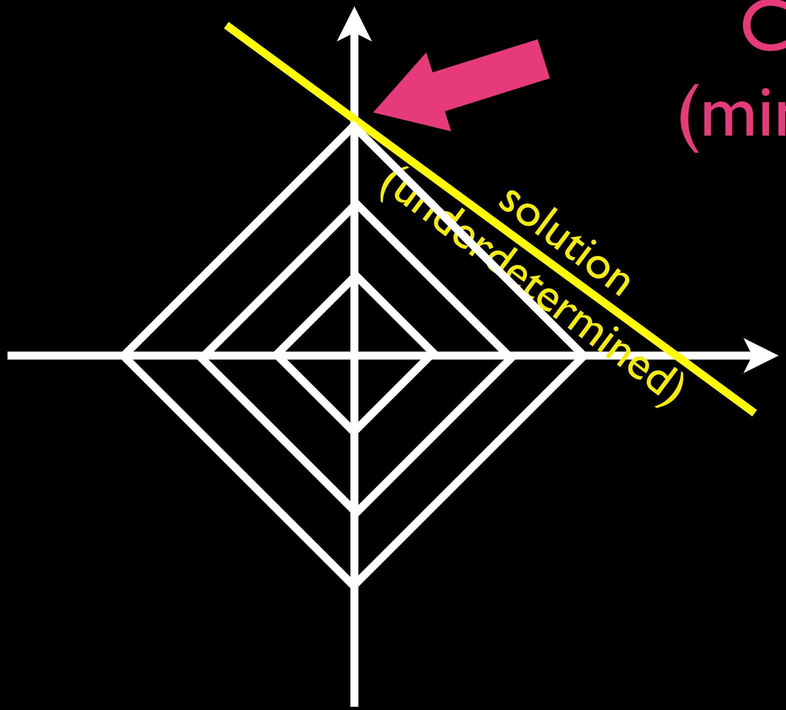
Least squares solution
(minimizes L2 on the line)

$$\begin{pmatrix} 2 & 4 \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} = \begin{pmatrix} 8 \end{pmatrix}$$

$$\text{s.t.} \quad c^2 + s^2 = \min$$

NOT Sparse

L1



Compressive solution
(minimizes L1 on the line)

$$\begin{pmatrix} 2 & 4 \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} = \begin{pmatrix} 8 \end{pmatrix}$$

$$\text{s.t.} \quad |c| + |s| = \min$$

Sparse

Generally, of the form $(0, \ldots, 0, \xi, 0, \ldots, 0)$

Sparse spiky signals

$$f(t) = \sum_{\tau \in T} f(\tau)\,\delta(t-\tau)$$

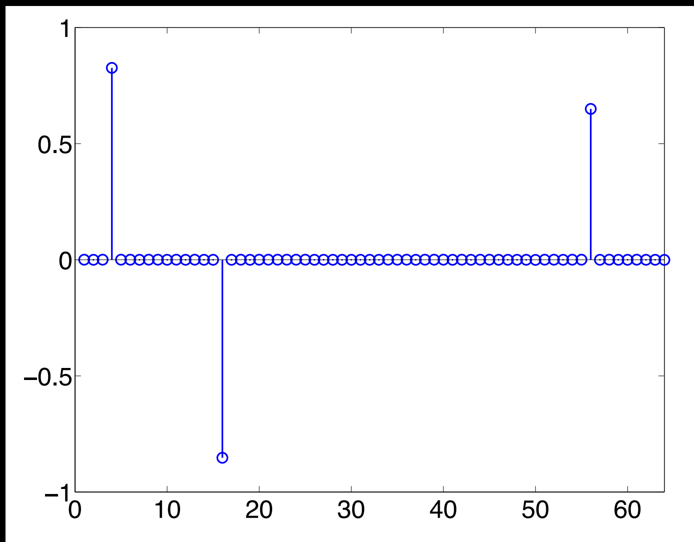
$$\hat{f}(w) = \sum_{x=0}^{N-1} f(x)\, e^{i2\pi w x/N}$$

$$\hat{f}(w) = \sum_{\tau \in T} f(\tau)\, e^{-i2\pi w \tau}$$

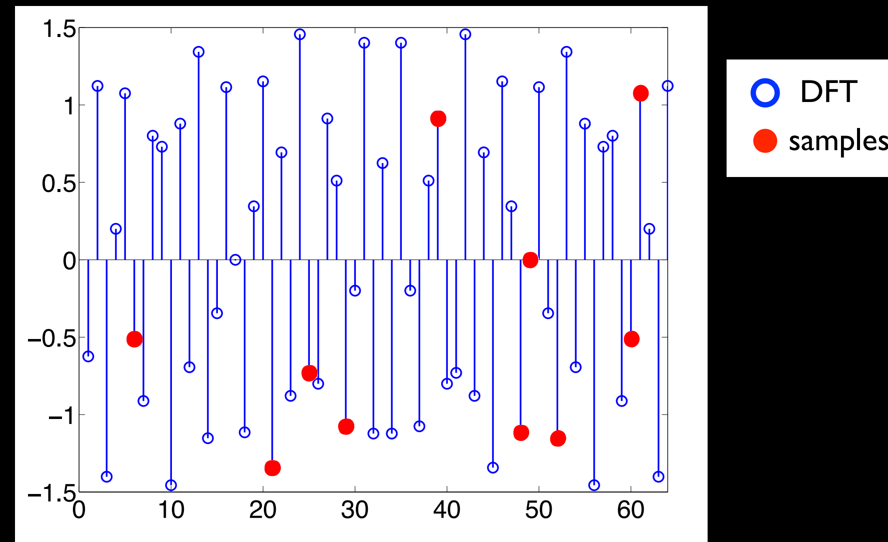
- ➡ Native space: spiky signal ⇒ Nyquist sampling necessary
- ➡ Fourier space: smooth signal (superposition of a few sinusoids only) ⇒ fewer than Nyquist samples perhaps suffice
- ➡ To make up for missing samples: L1 minimization

Compressive sensing example

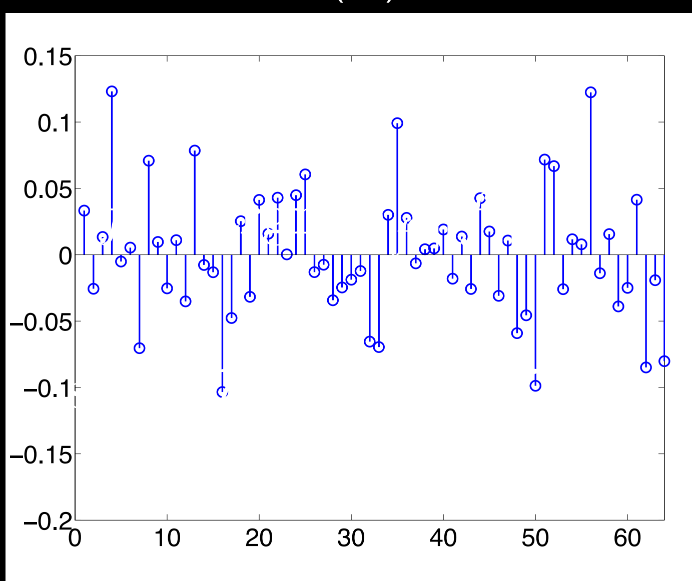
Original signal with 3 spikes (total length=64)



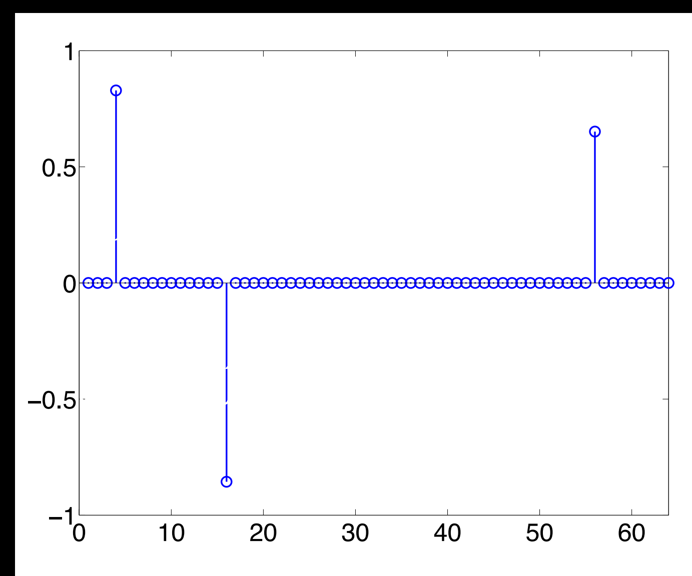
DFT measurements (# of samples=12)



Conventional (L2) reconstruction



Compressive (L1) reconstruction



Reconstruction success is subject to sparsity

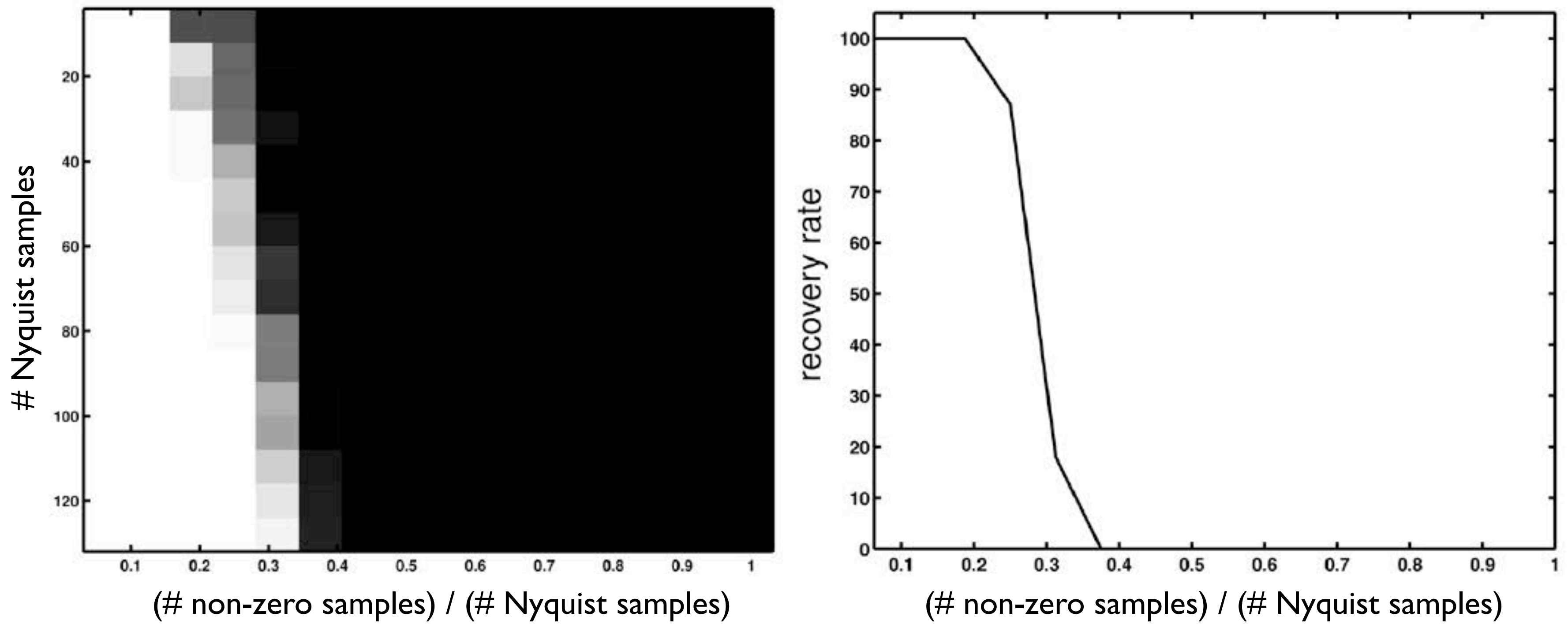


Fig. 2. Recovery experiment for $N = 512$. (a) The image intensity represents the percentage of the time solving (P_1) recovered the signal f exactly as a function of $|\Omega|$ (vertical axis) and $|T|/|\Omega|$ (horizontal axis); in white regions, the signal is recovered approximately 100% of the time, in black regions, the signal is never recovered. For each $|T|$, $|\Omega|$ pair, 100 experiments were run. (b) Cross section of the image in (a) at $|\Omega| = 64$. We can see that we have perfect recovery with very high probability for $|T| \leq 16$.

E. Candés, J. Romberg, and T. Tao, *IEEE Trans. Info. Th.* 52:489, 2006

The premise of compressive sensing

- Nyquist criterion is too restrictive because it takes no priors into account
- Most signals are ***sparse*** if expressed in an appropriate basis, e.g.
  - sparse in time - “spiky”
  - sparse in frequency - “beaty”
- Far ***fewer*** samples than Nyquist may suffice to completely reconstruct, provided
  - the appropriate basis has been selected
  - sufficient ***signal mixing*** by the measurement operator
    - “measurement must be ***incoherent***”
- Sparsity can then be enforced as a prior (regularizer) by $\mathcal{L}1$ norm minimization

E. Candès, J. Romberg, and T. Tao, IEEE Trans. Info. Th. 52:489, 2006
D. Donoho, IEEE Trans. Info. Th., 52:289, 2006
E. Candès and T. Tao, IEEE Trans. Info. Th., 52: 5406, 2006

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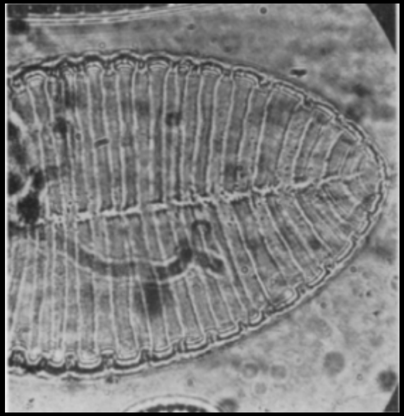
The significance of phase

Visible

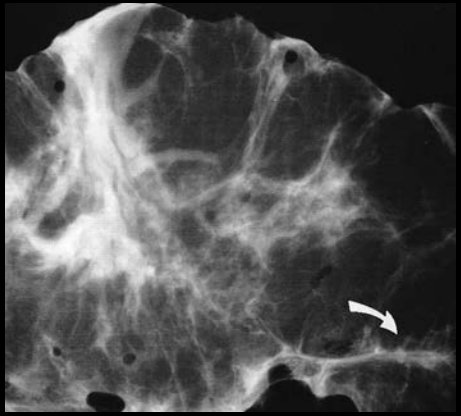


intensity image

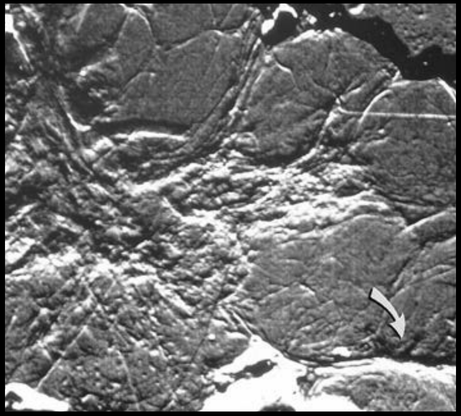
phase-contrast image

(F. Zernike, Science 121, 1955)

X-ray



attenuation image



phase-contrast image

(human breast cancer specimen)

(E. D. Pisano et al., Radiology 214, 2000)

$$\phi(x_o) = k \int_\Gamma n(\mathbf{r})\mathrm{d}l \;\Rightarrow\; \rho \propto \frac{n^2 - 1}{n^2 + 2}$$

Refractive index

Density

$\Rightarrow \rho \propto$ temperature, pressure, humidity

Phase is irrelevant!

Optical Path Length (OPL)

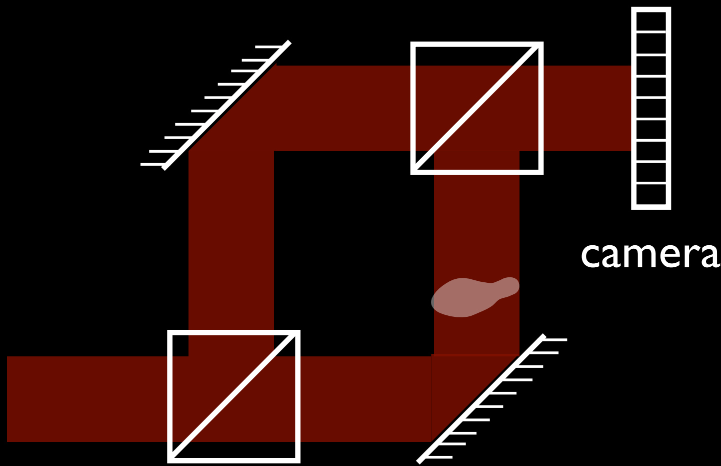
is relevant

(and works with partially coherent light)

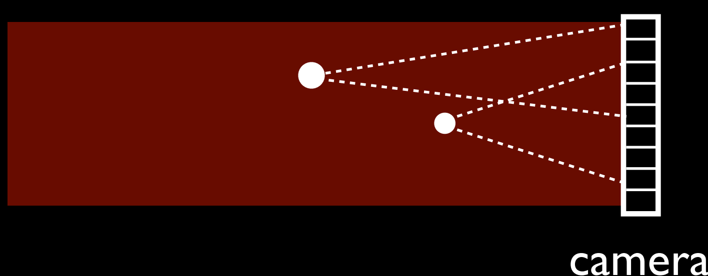
J. C. Petruccelli, et al, *Opt. Express* 21:14430, 2013

Phase Imaging

- Interferometric

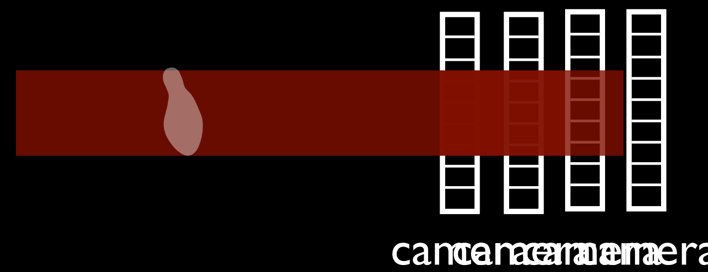


External reference



Self-referenced (in-line digital holography)

- Axial stack



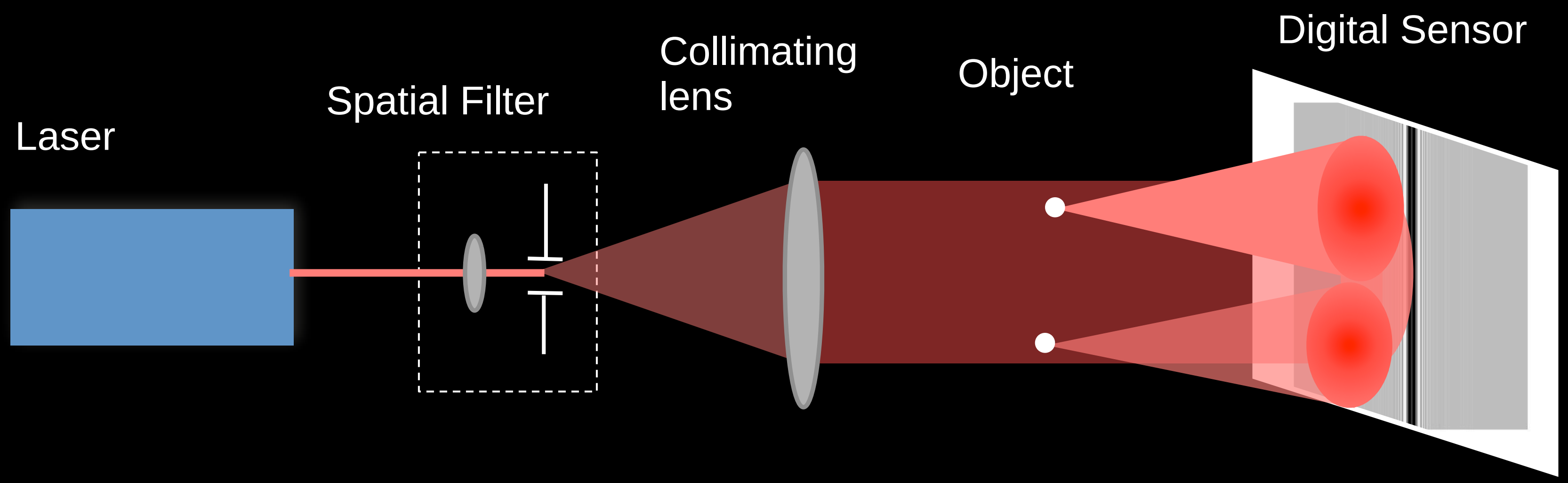
measuring phase (OPL)

TODAY

object/particle localization

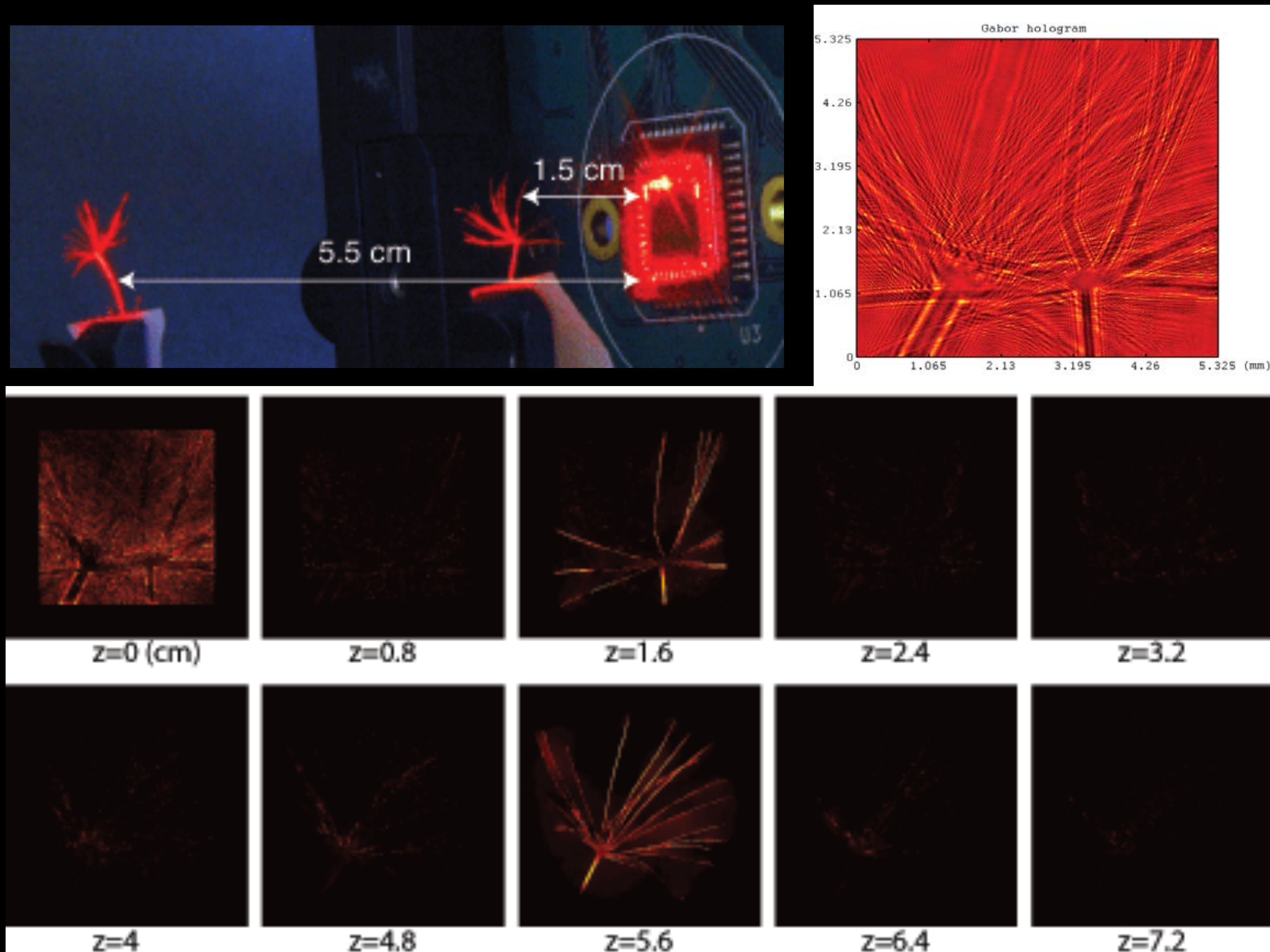
Digital Holography:

Measurement is "incoherent" !



- ➡ According to <u>Statistical Optics</u>, a digital hologram is formed by interfering spatially and temporally *coherent* beams (e.g. originating from a HeNe laser)
- ➡ According to <u>Compressive Sensing</u> theory, this measurement is "*incoherent*" because light scattered from the object spreads out over several pixels

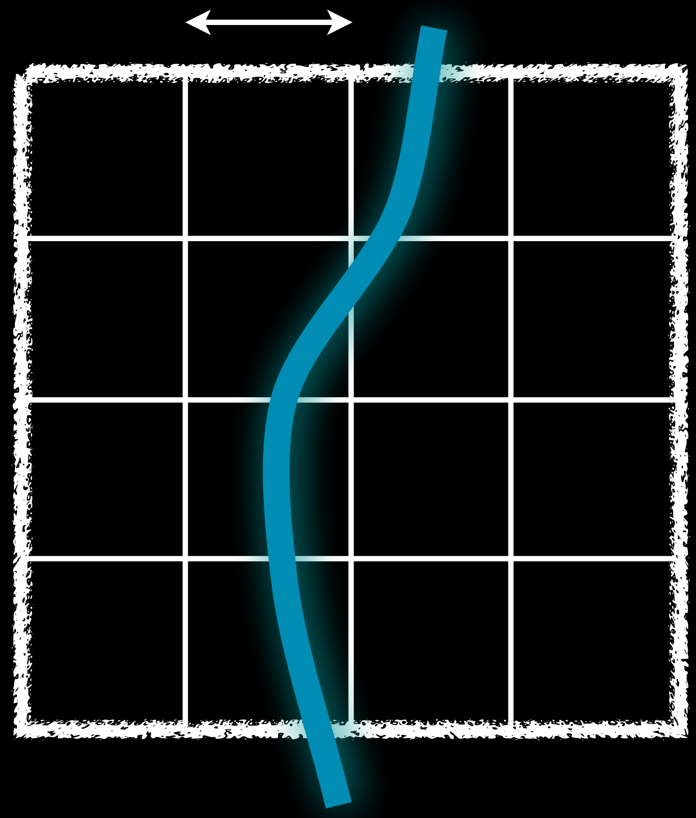
Compressive Holography



D. J. Brady, et al *Opt. Express* 17:13040, 2009.

Compressive localization

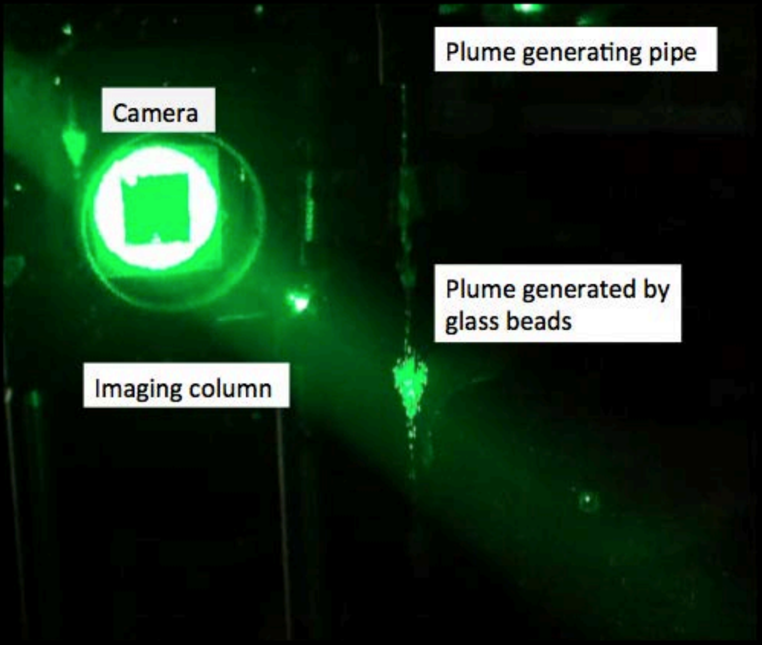
Desirable accuracy: < 1 pixel



Prior: sparsity of object(s) within the field of view

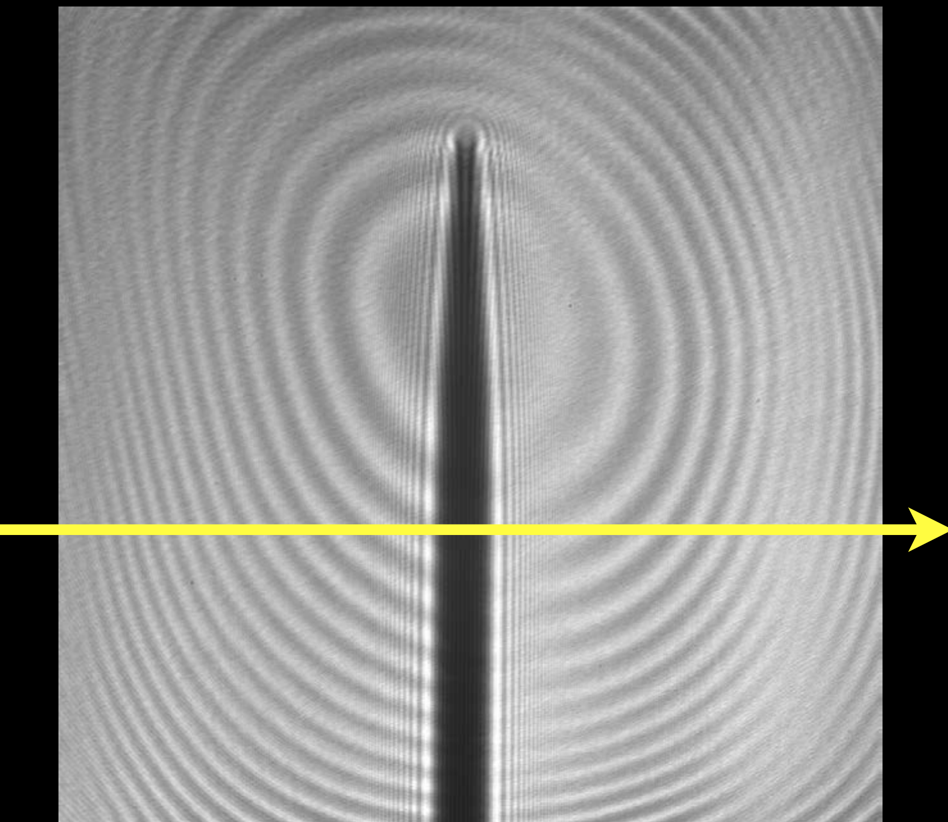
Localization examples

Quantitative measurement
of seal whisker motion

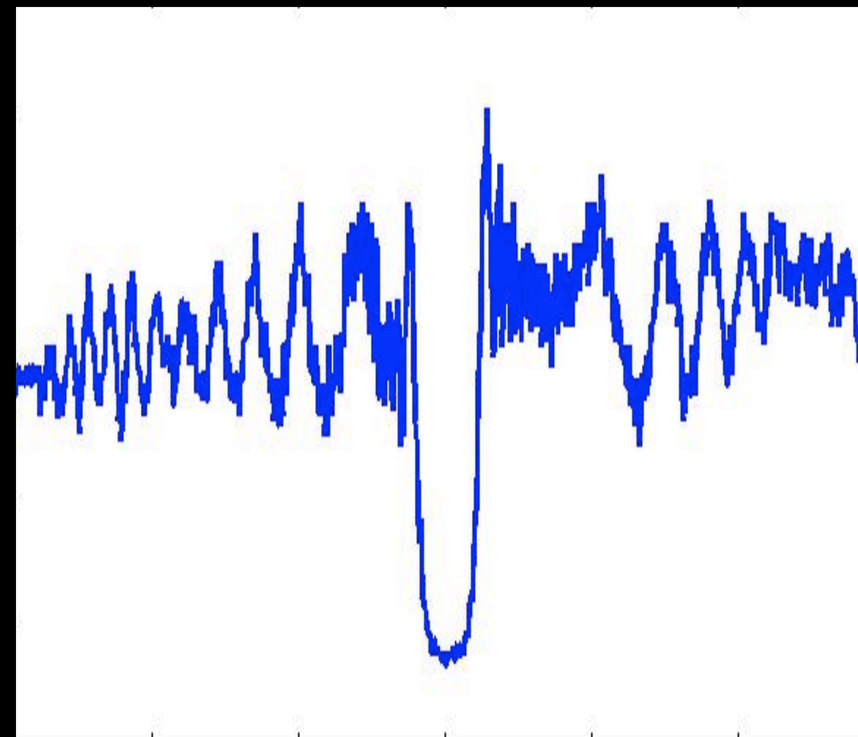


Quantitative analysis
of bubbles and plumes
(multi-phase flows)

Emulating a 1D whisker: pin object

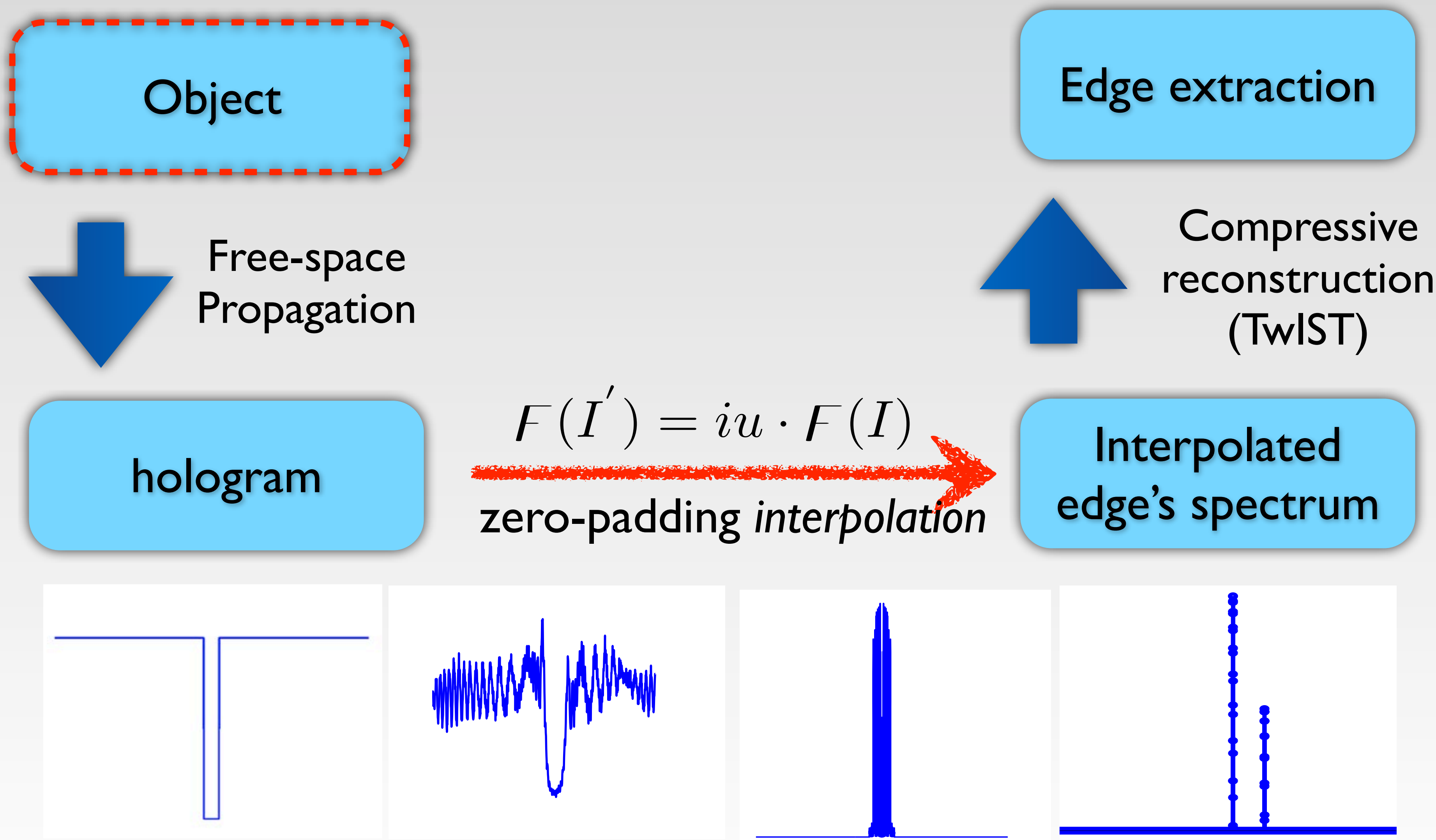


Digital hologram across 1 row

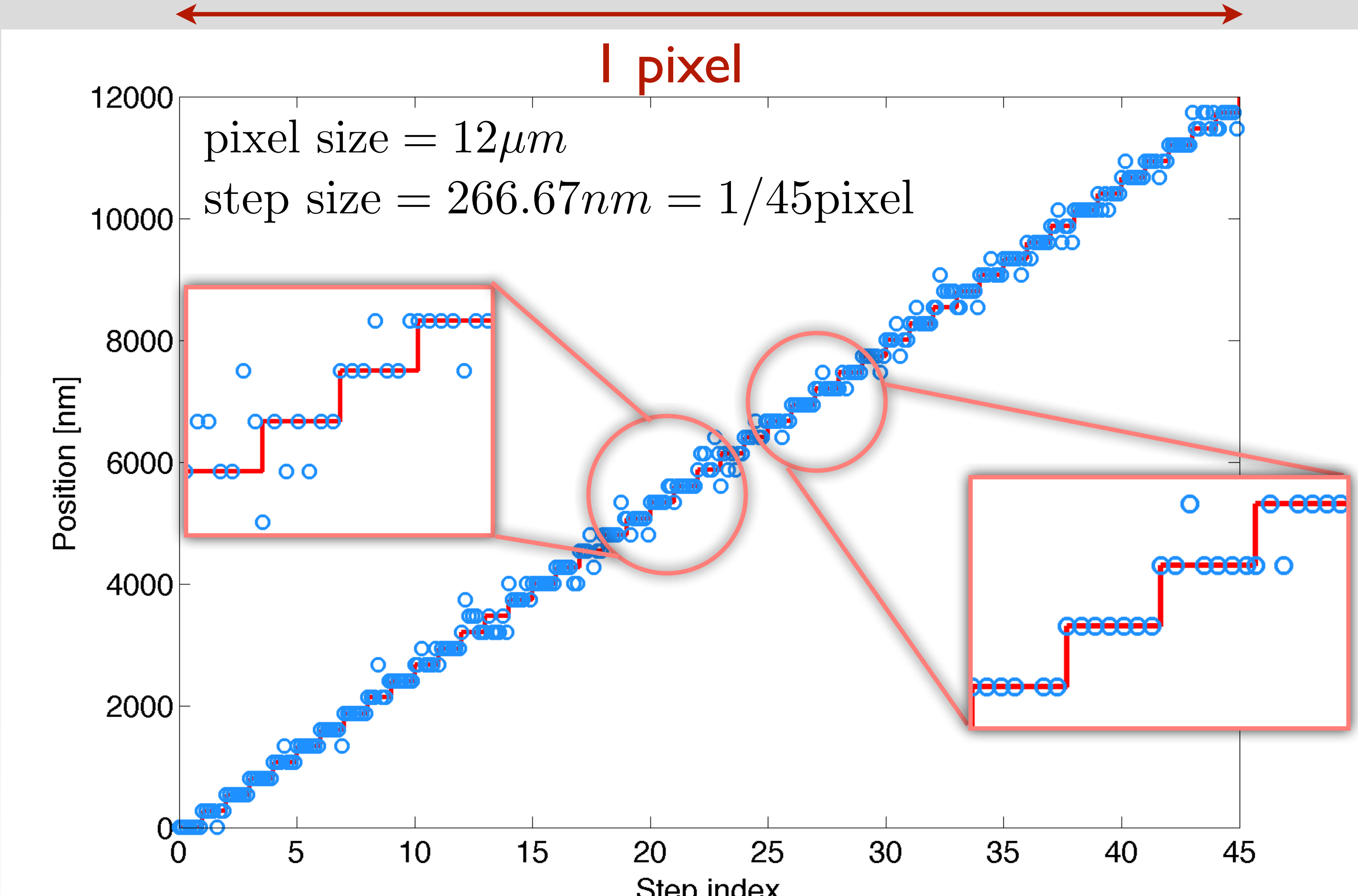


Yi Liu et al, *Opt. Lett.* 37:3357, 2012

Algorithm diagram



Experimental result

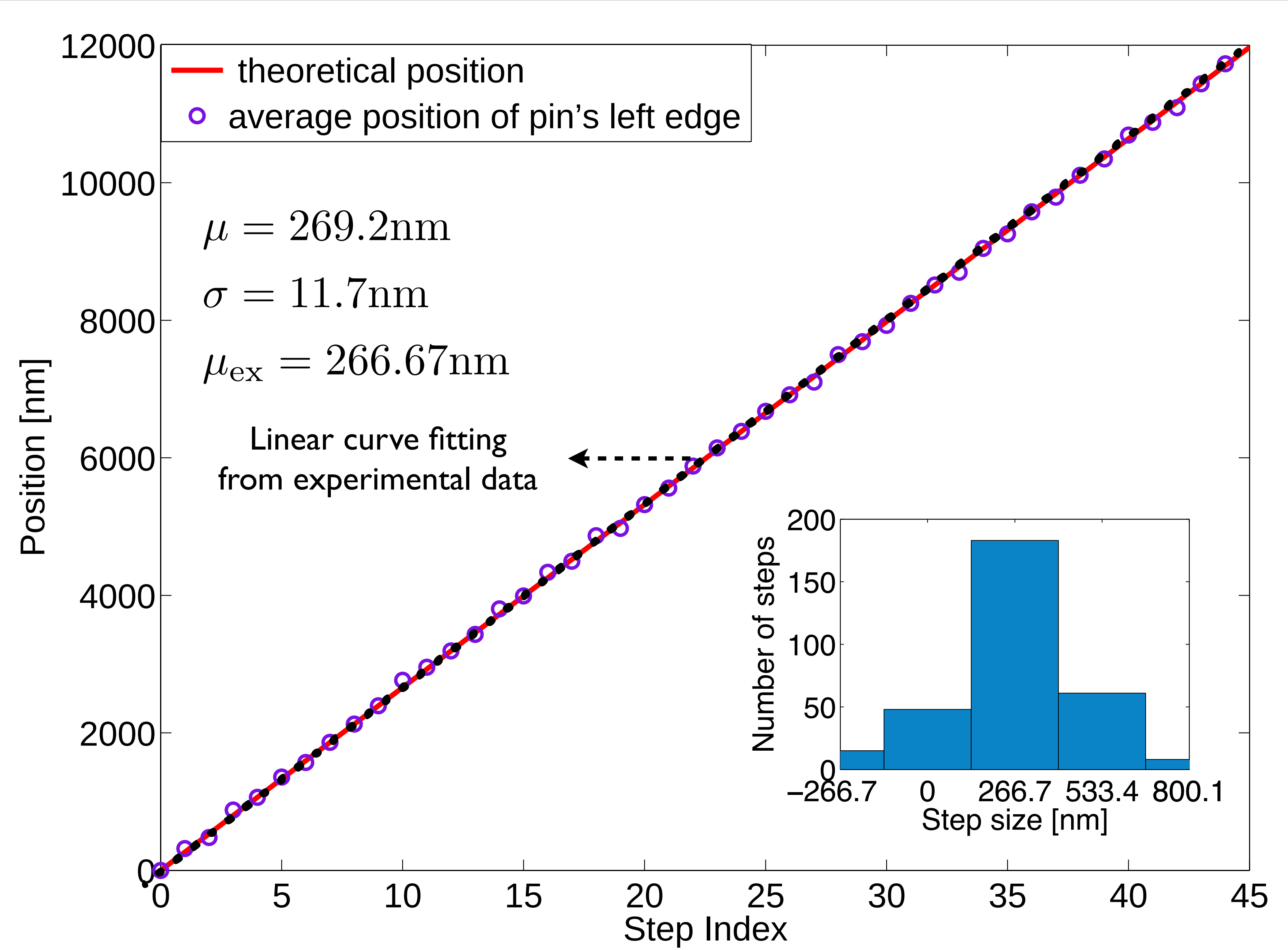


○ : position of the left point of one row on the pin at each step
Each step, tracing 7 rows.

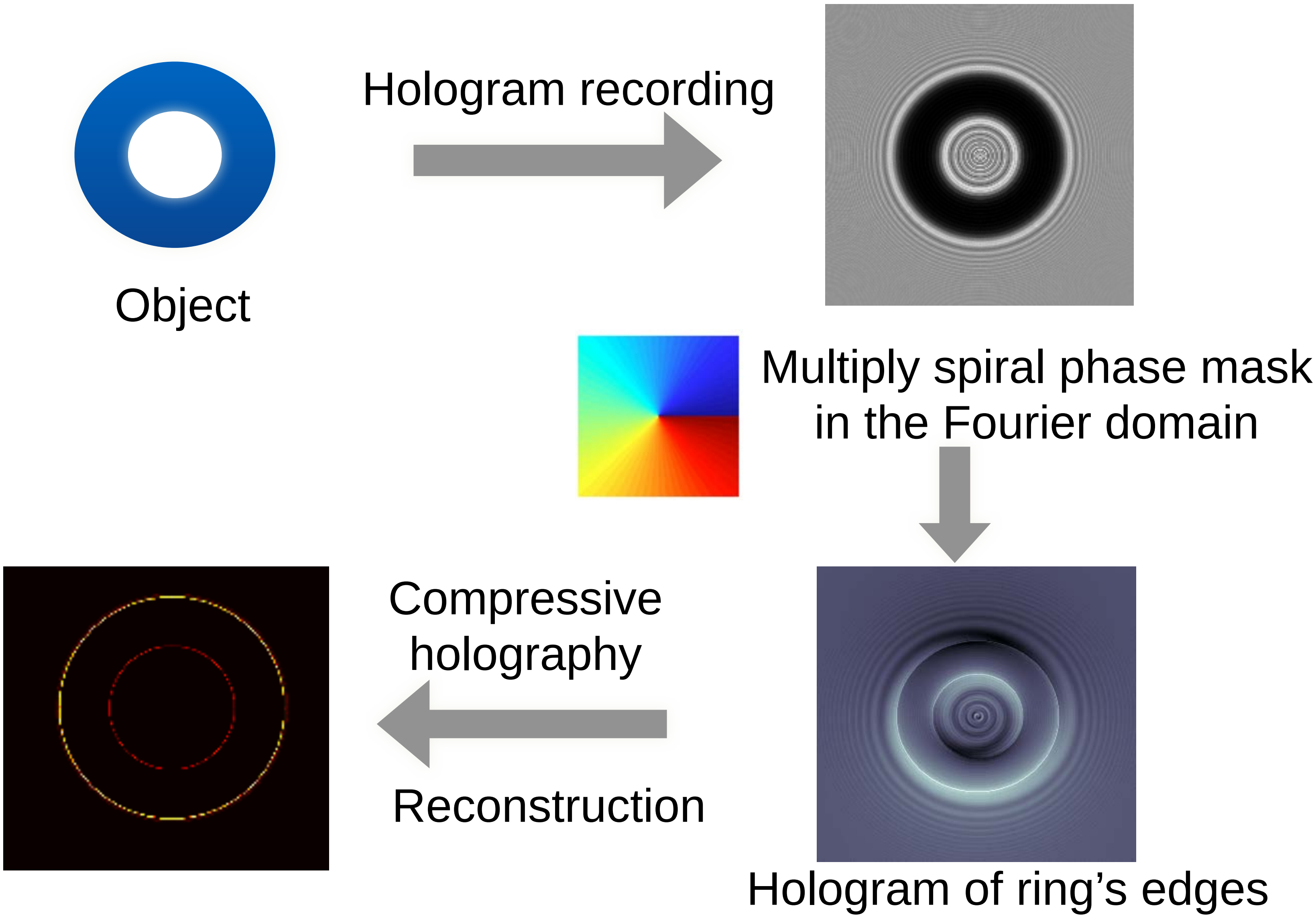
Yi Liu et al, to appear in *Optics Letters* issue August 15, 2012

Experimental result

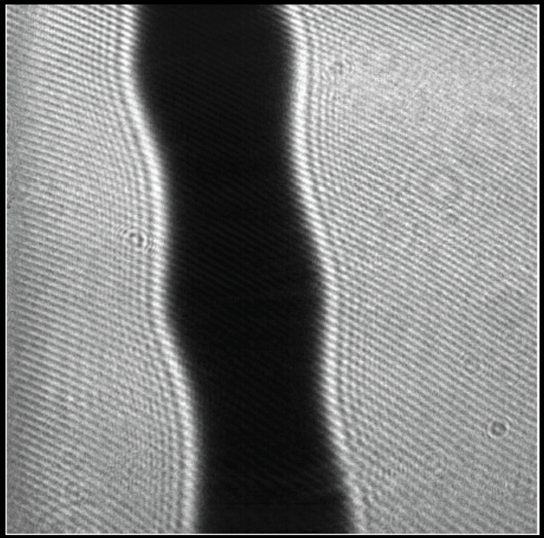
`



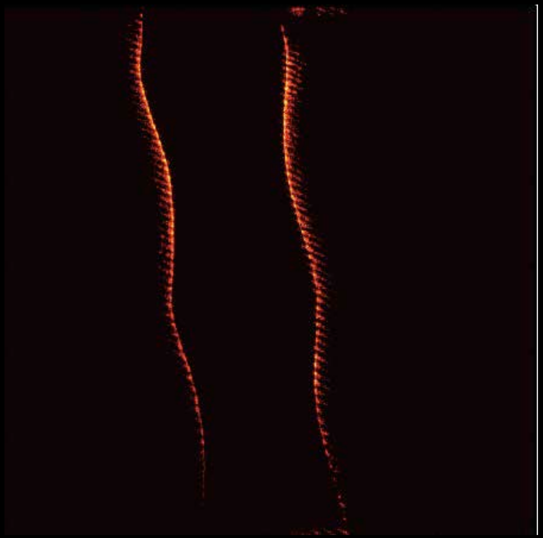
2D object localization



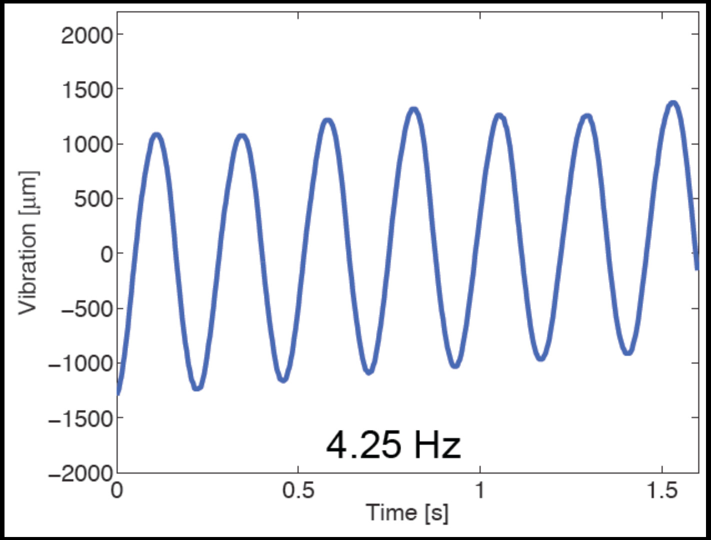
Whisker vibration experiments



Whisker hologram



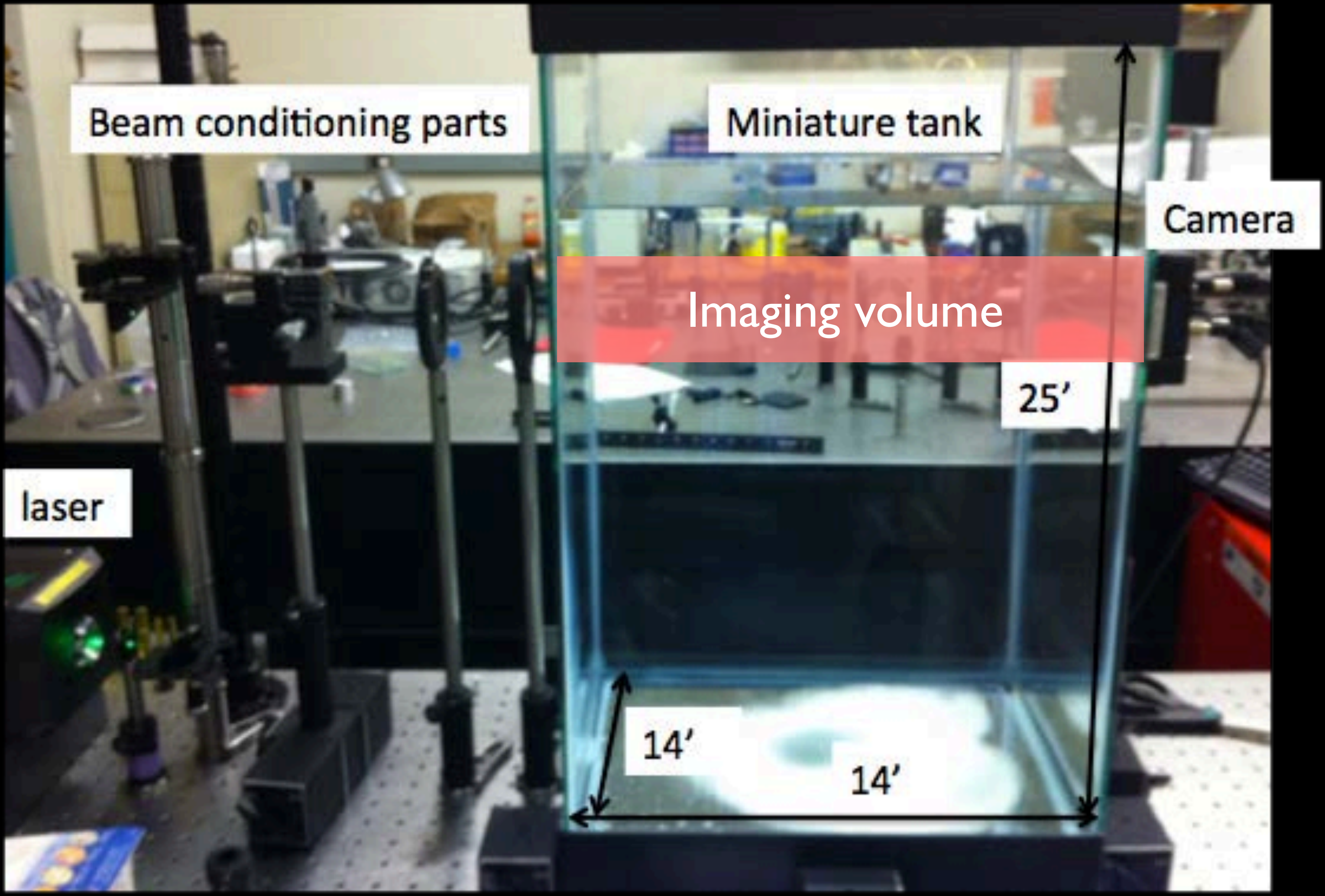
Whisker's edges extracted



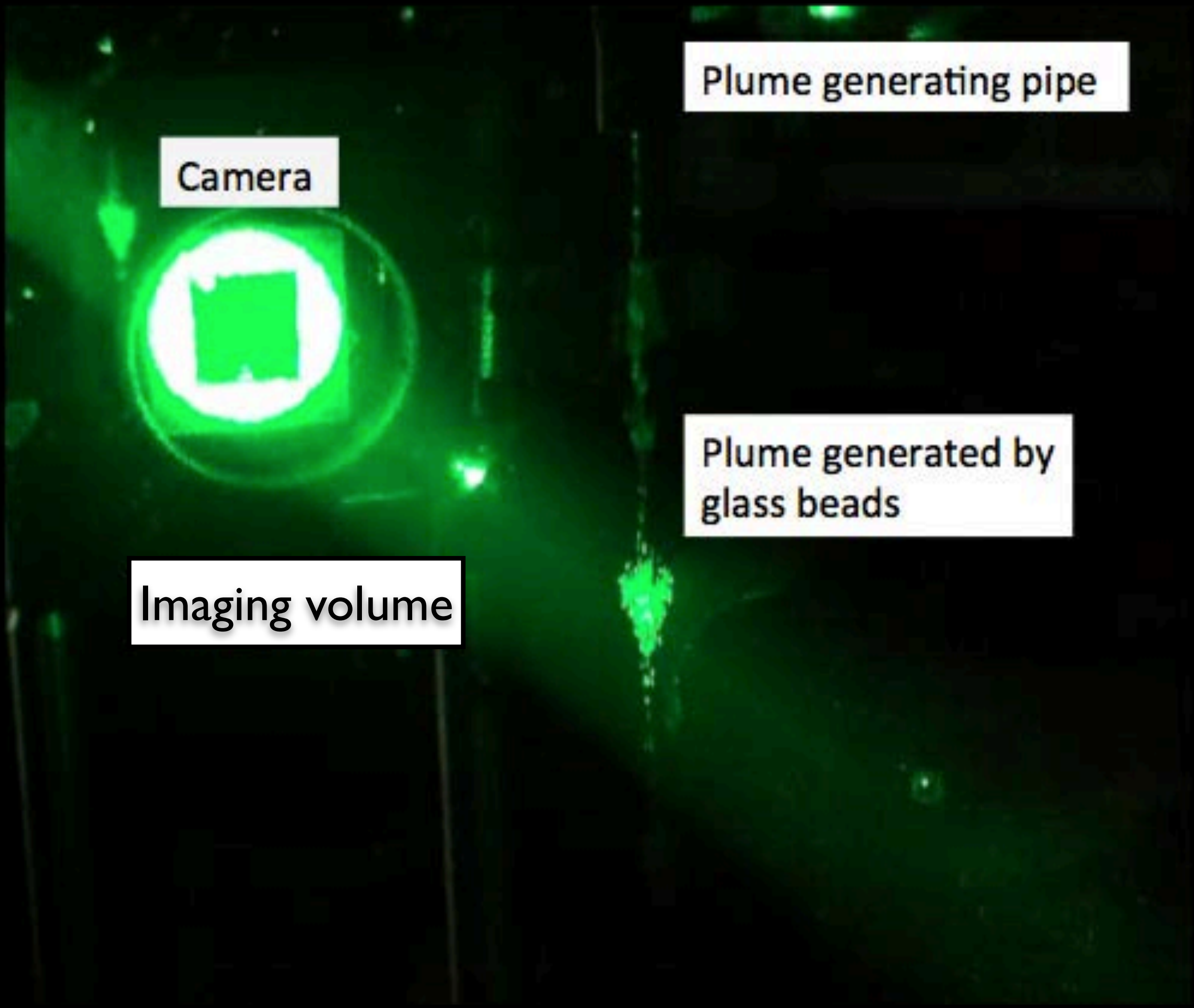
Whisker's motion reconstructed (pixel size = 10μm

Acknowledgment
Heather Beem,
Michael Triantafyllou
MIT

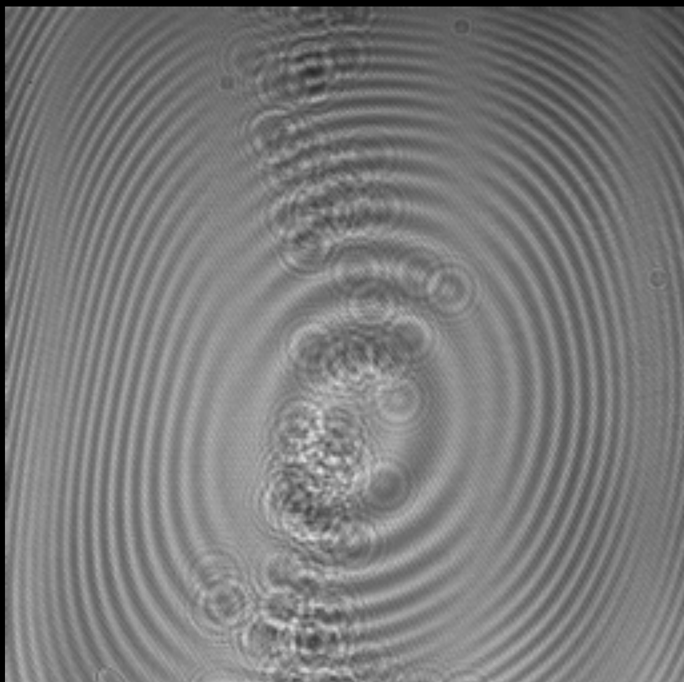
DH particle localization



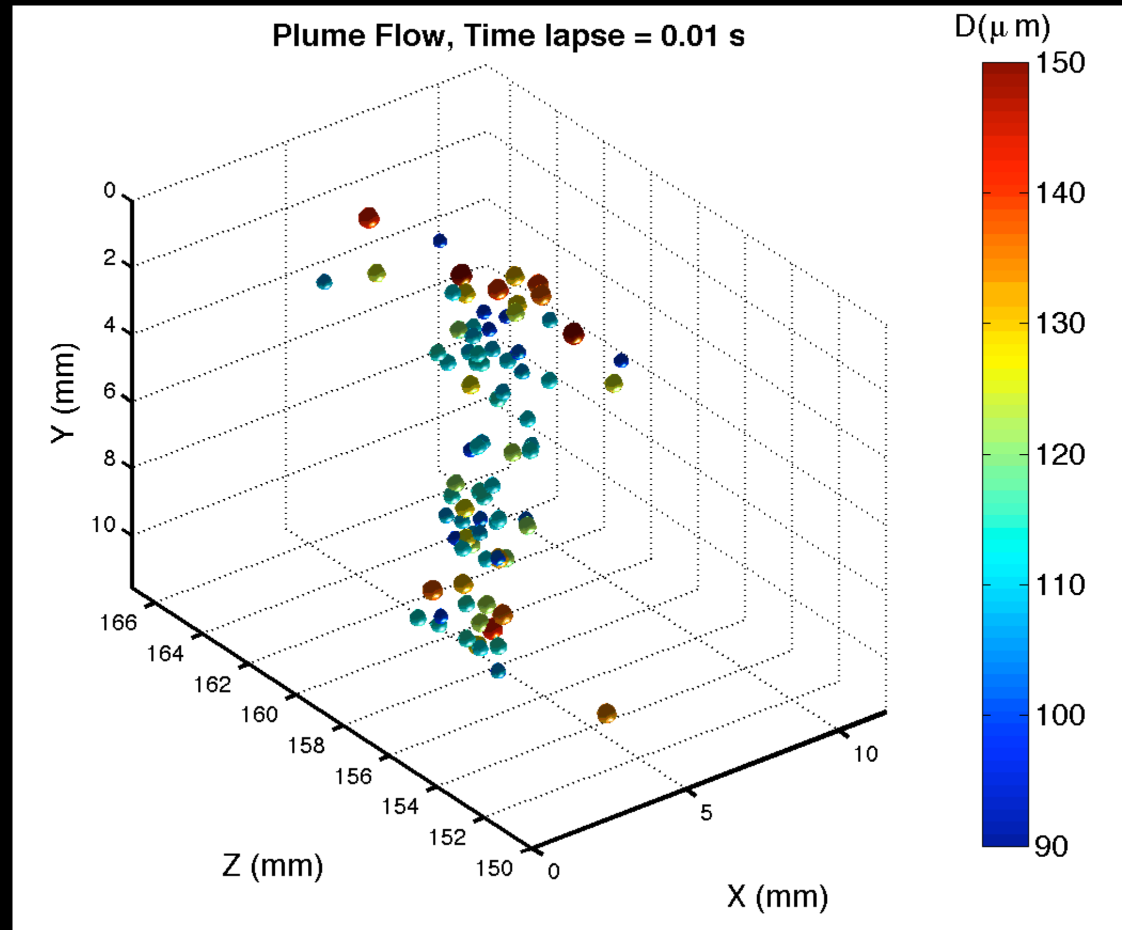
DH particle localization



3D reconstruction of a plume (standard back-propagation)



Original hologram

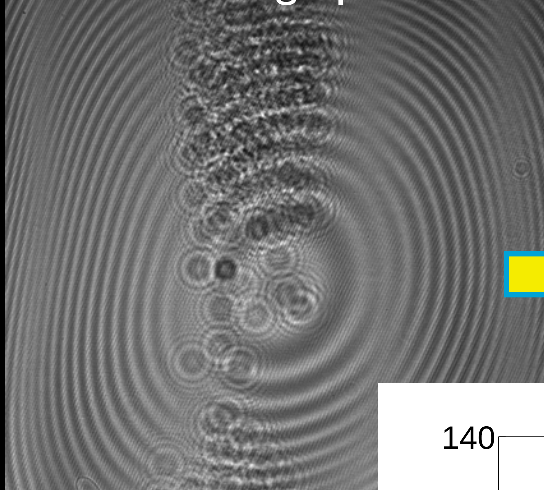


3D Reconstruction

Particle size not to scale

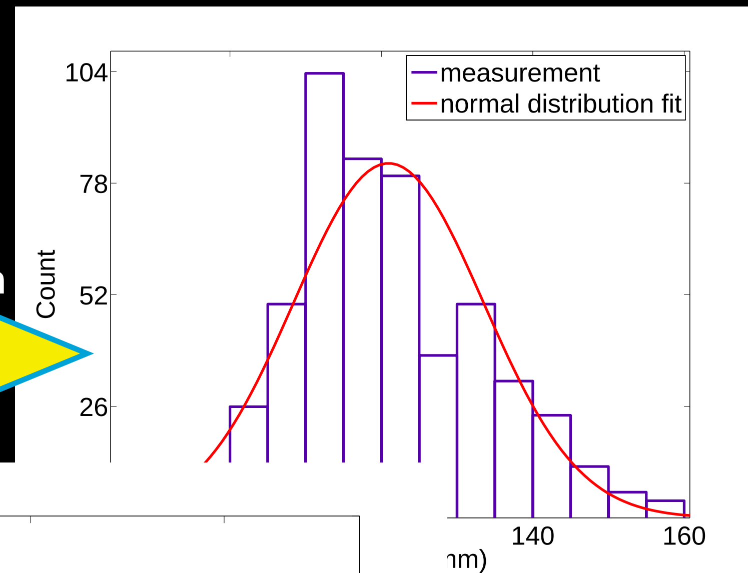
Size distribution analysis

a single frame
from the holographic movie

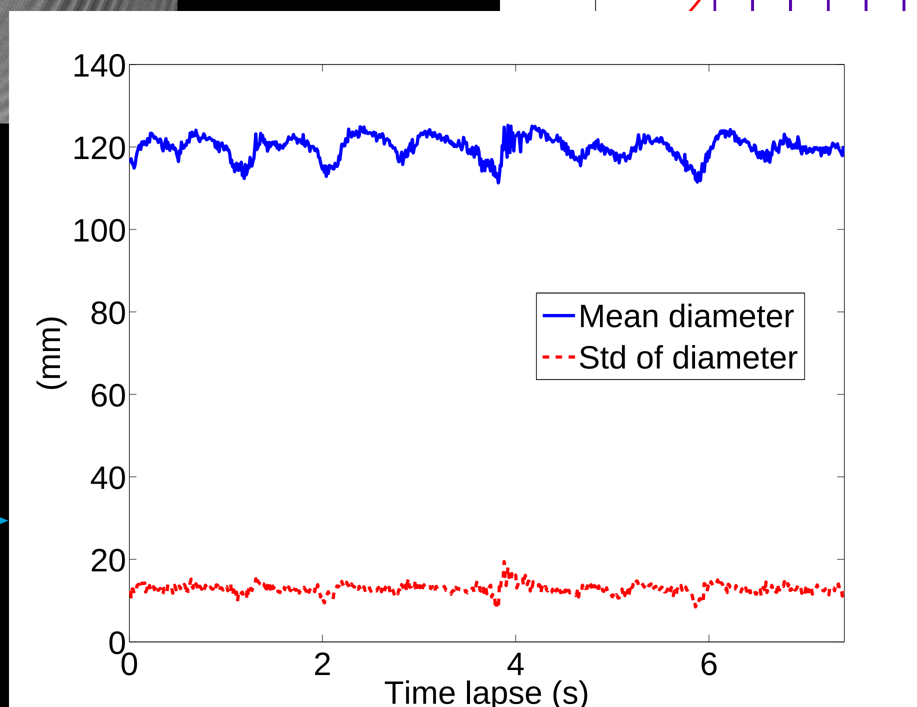


Reconstruction

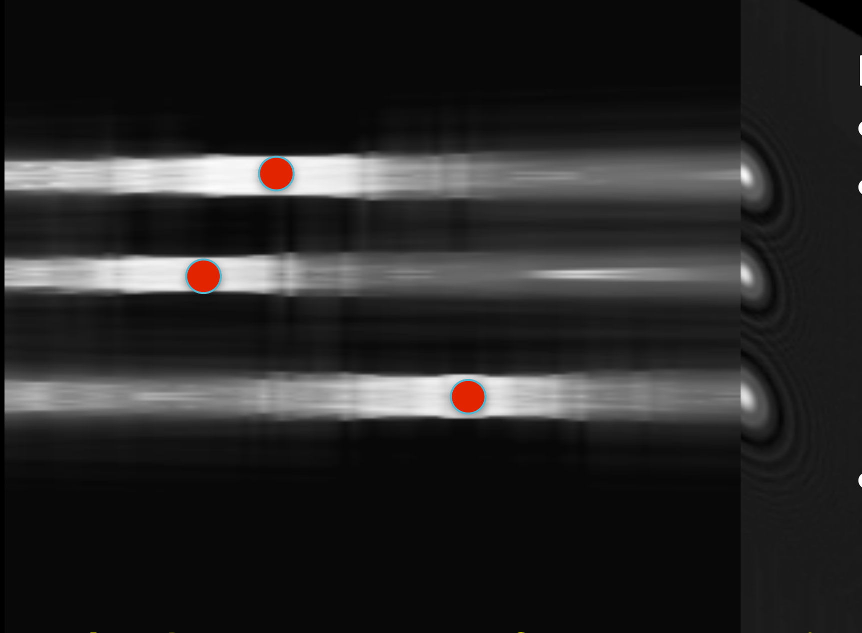
size distribution



repeat for
each frame



Sharpening the axial accuracy



In a given *z* plane:

- Sharp features mostly due to in-focus particles
- Smooth features due to:
    - Defocused particles
    - Twin image
    - Halo
    - Noise
- This essentially states a sparsity prior on the edge sharpness

In this case we enforce **sparsity** by evolving the unknown radiance *f* to the steady state of a nonlinear diffusion equation

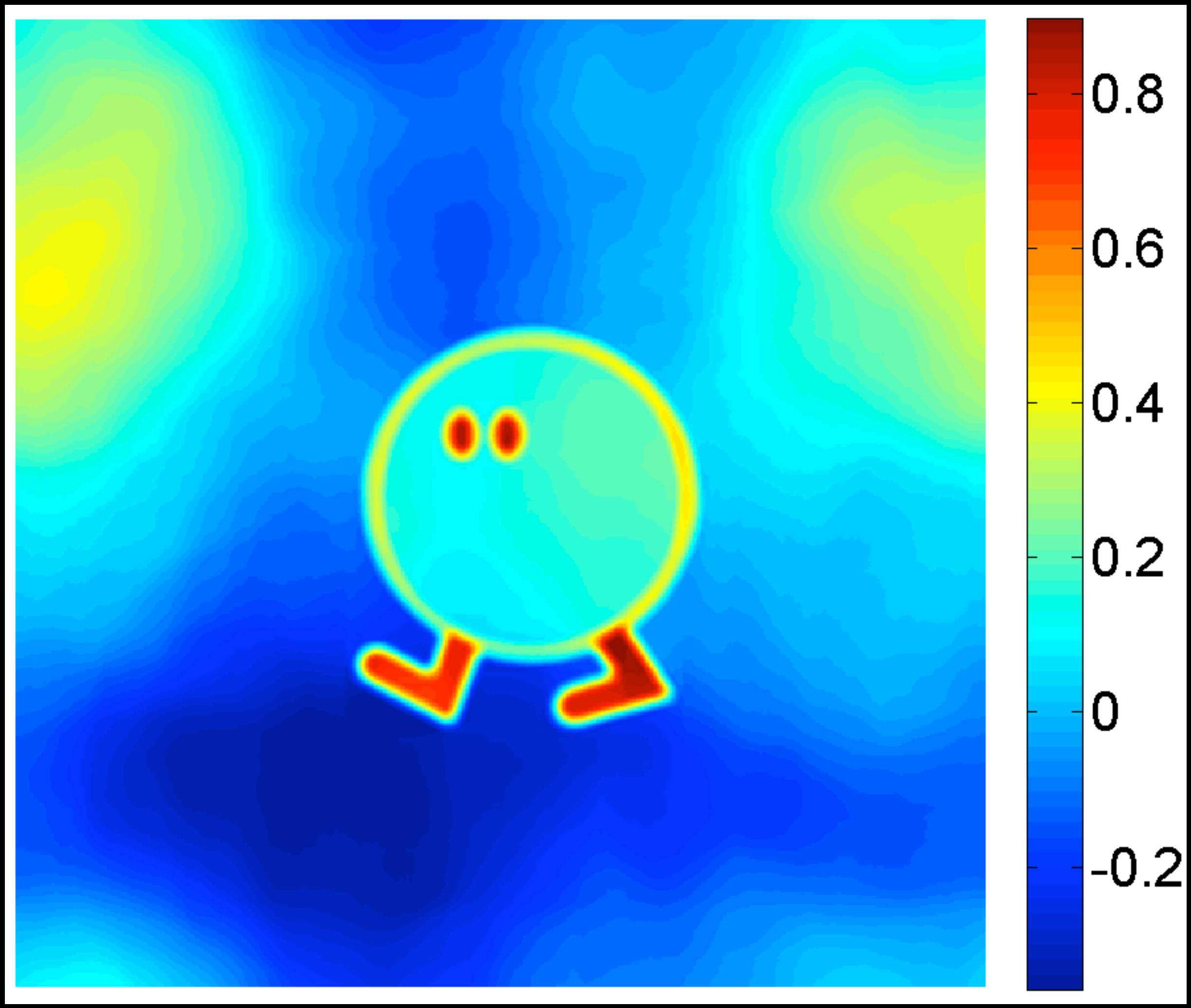
$$\frac{\partial f\left(\mathbf{x}, z; \tau\right)}{\partial \tau} = \alpha \nabla \cdot \left(F\left(\left|\nabla f\right|\right) \frac{\nabla f}{\left|\nabla f\right|} \right)$$

F: flux function

(notice $F=1 \Rightarrow$ linear diffusion)

L. Tian, J. C. Petruccelli, and G. Barbastathis, *Opt. Lett.* 37:4131, 2012.

Nonlinear diffusion: animation

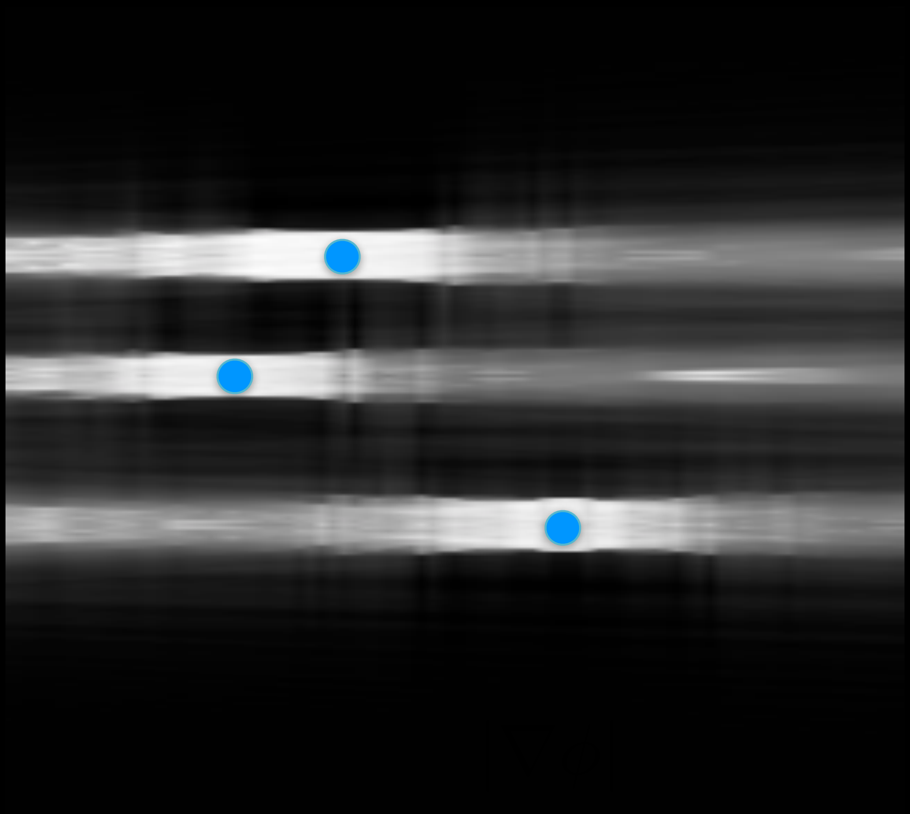


(movie shows output at every iteration)

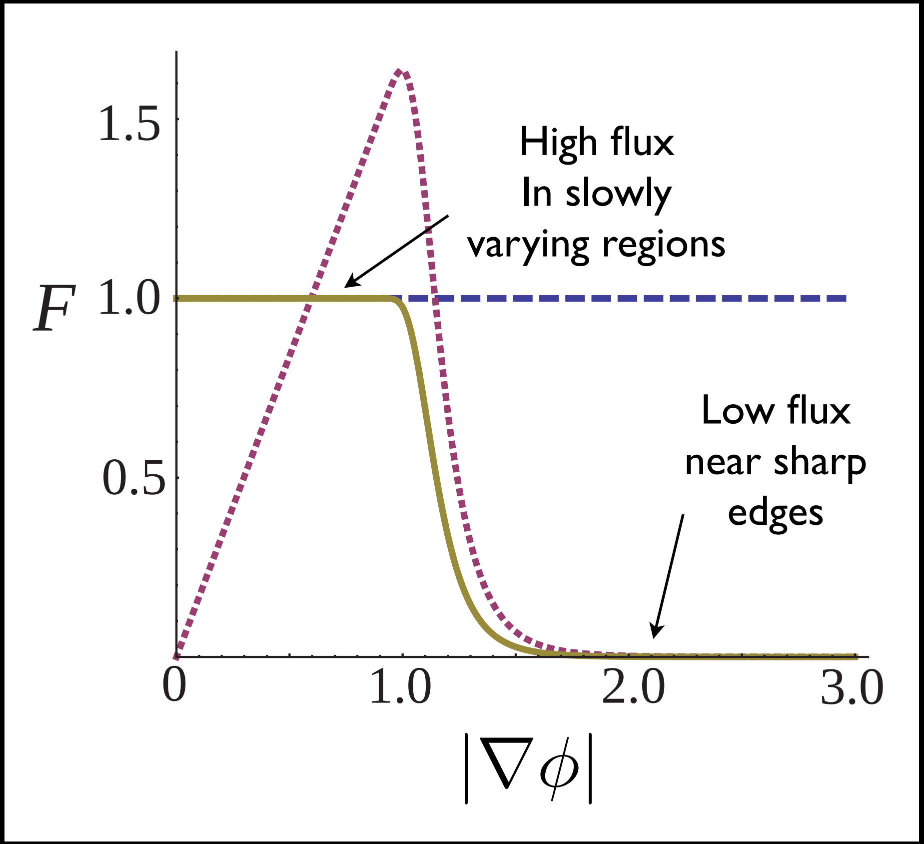
L. Tian, J. C. Petruccelli, and G. Barbastathis, *Opt. Lett.* 37:4131, 2012.

Sharpening the axial accuracy by nonlinear diffusion

NLD in the *transverse* direction



Flux



L. Tian, J. C. Petruccelli, and G. Barbastathis, *Opt. Lett.* 37:4131, 2012.
J. Weickert, *Lecture Notes in Computer Science*, 1252:1, 1997.

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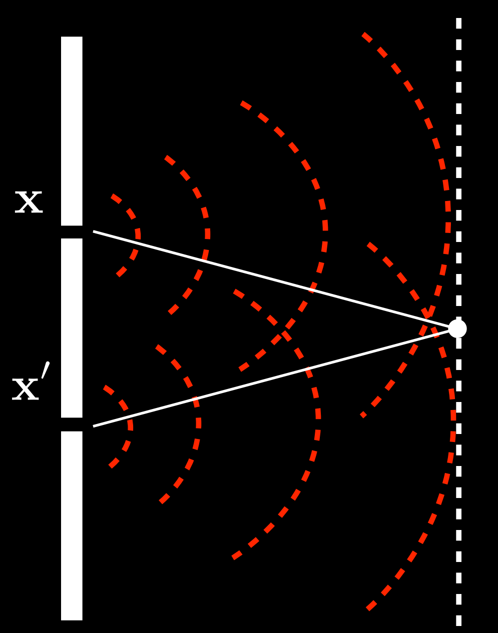
Partially coherent light

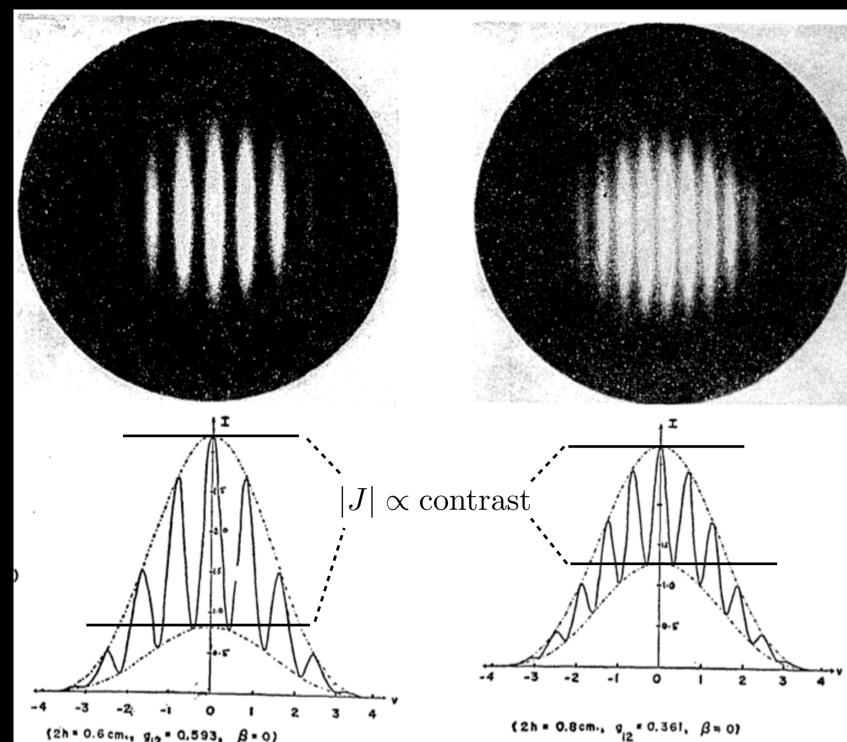
Random field $U(\mathbf{x})$

Correlation function (mutual intensity)

$$J(\mathbf{x}, \mathbf{x}') \equiv \langle U(\mathbf{x}) U^*(\mathbf{x}') \rangle$$



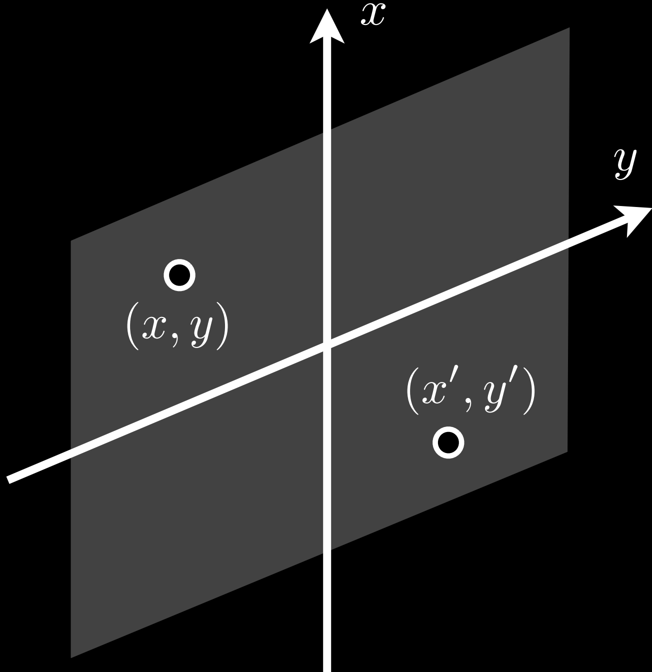
Young's two-slit experiment

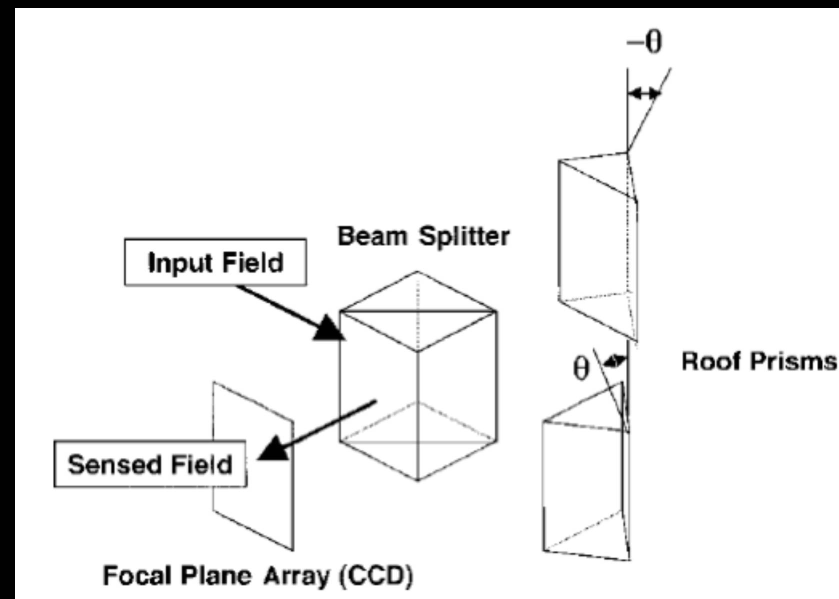


B. J. Thompson and E. Wolf, *J. Opt. Soc. Am.*, 47:895, 1957.

The mutual intensity

$$J(\mathbf{x},\mathbf{x}') \equiv \langle U(\mathbf{x})U^*(\mathbf{x}')\rangle$$





D. L. Marks, R. A. Stack, and D. Brady, *Appl. Opt.* 38:1332, 1999

The mutual intensity

$$J(\mathbf{x}, \mathbf{x}') \equiv \langle U(\mathbf{x}) U^*(\mathbf{x}') \rangle$$

- completely characterizes the (quasi-monochromatic) partially coherent field,
- in particular, the Optical Path Length (OPL);
  - J. C. Petruccelli, L. Tian, and G. Barbastathis, *Opt. Express* 21:14430, 2013
- is analogous to the density matrix in quantum mechanics;
- is semi-positive definite (eigenvalues≥0);
- can be decomposed into **coherent modes**

$$J\left(\mathbf{x}, \mathbf{x}'\right) = \sum_j c_j \, \phi_j\left(\mathbf{x}\right) \phi_j^*\left(\mathbf{x}'\right)$$

- often, a case can be made that only few $c_j \neq 0$

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The Phase Space

- Wigner distribution function

$$W(x,u) = \int \psi\left(x + \frac{x'}{2}\right)\psi^*\left(x - \frac{x'}{2}\right)\exp\left(-i2\pi u x'\right)\mathrm{d}x'$$

<.> < ... >

$$W(x,u) = \int J\left(x + \frac{x'}{2}, x - \frac{x'}{2}\right)\exp\left(-i2\pi u x'\right)\mathrm{d}x'$$

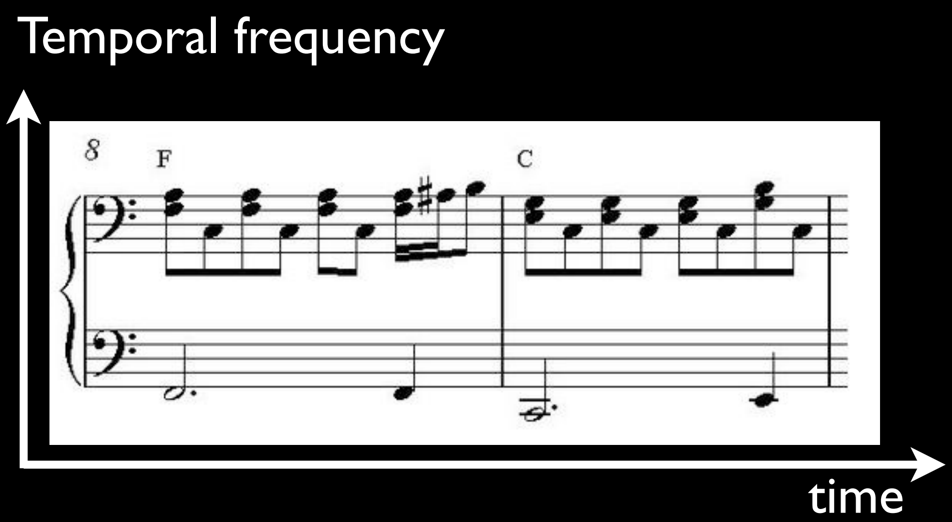
$$\Updownarrow \quad \mathcal{F}_{\substack{x \leftrightarrow u' \\ u \leftrightarrow x'}}$$

- Ambiguity function

$$A(u',x') = \int J\left(x + \frac{x'}{2}, x - \frac{x'}{2}\right)\exp\left(-i2\pi u' x\right)\mathrm{d}x$$

- By the way, $W(x,u)$ is real.

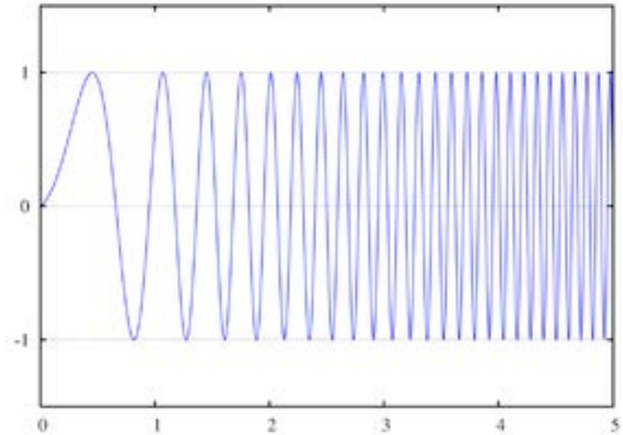
Phase space (Wigner space)



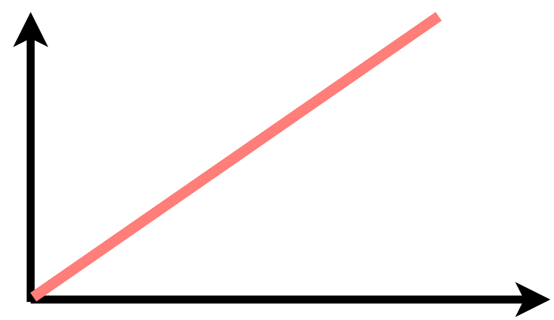
- Instantaneous frequency
- Local spatial frequency

Spherical wave

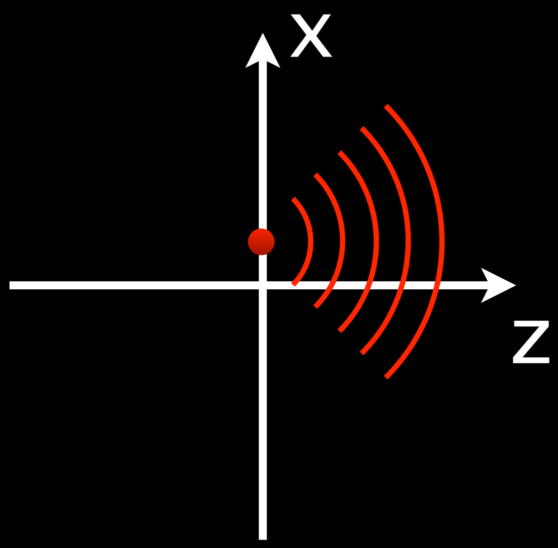
Chirp function



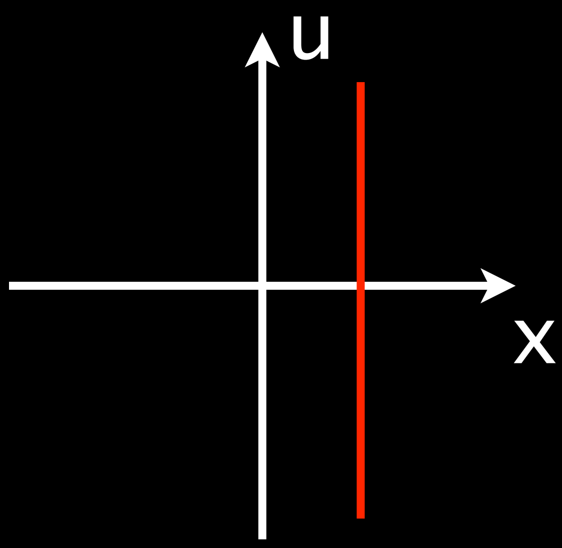
Phase space picture



point source

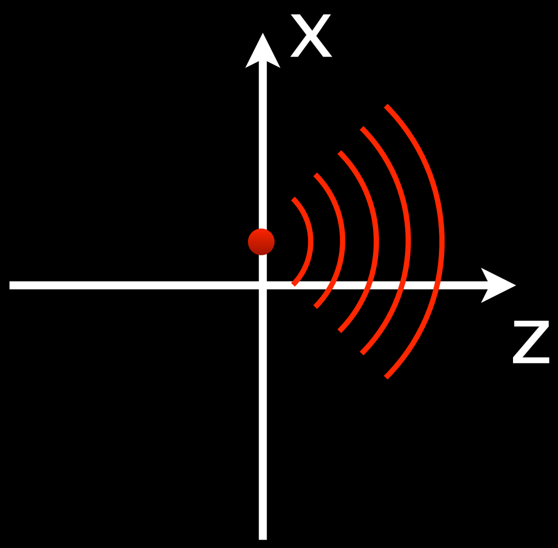


x-z space

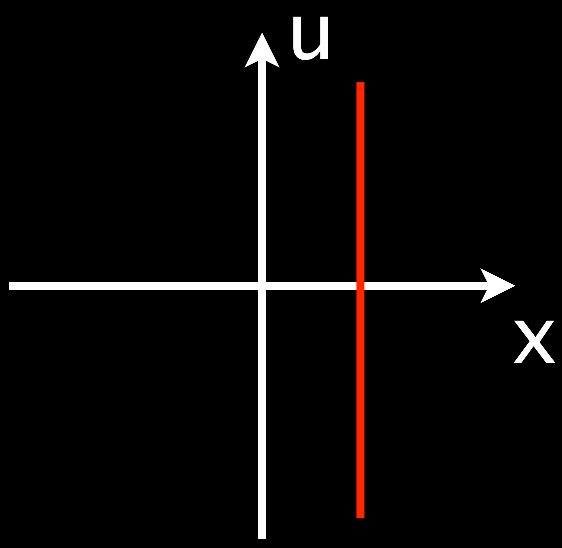


Wigner space
(x-u space)

spherical wave



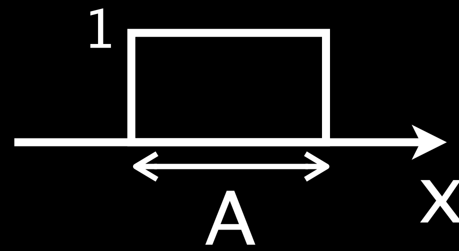
x-z space

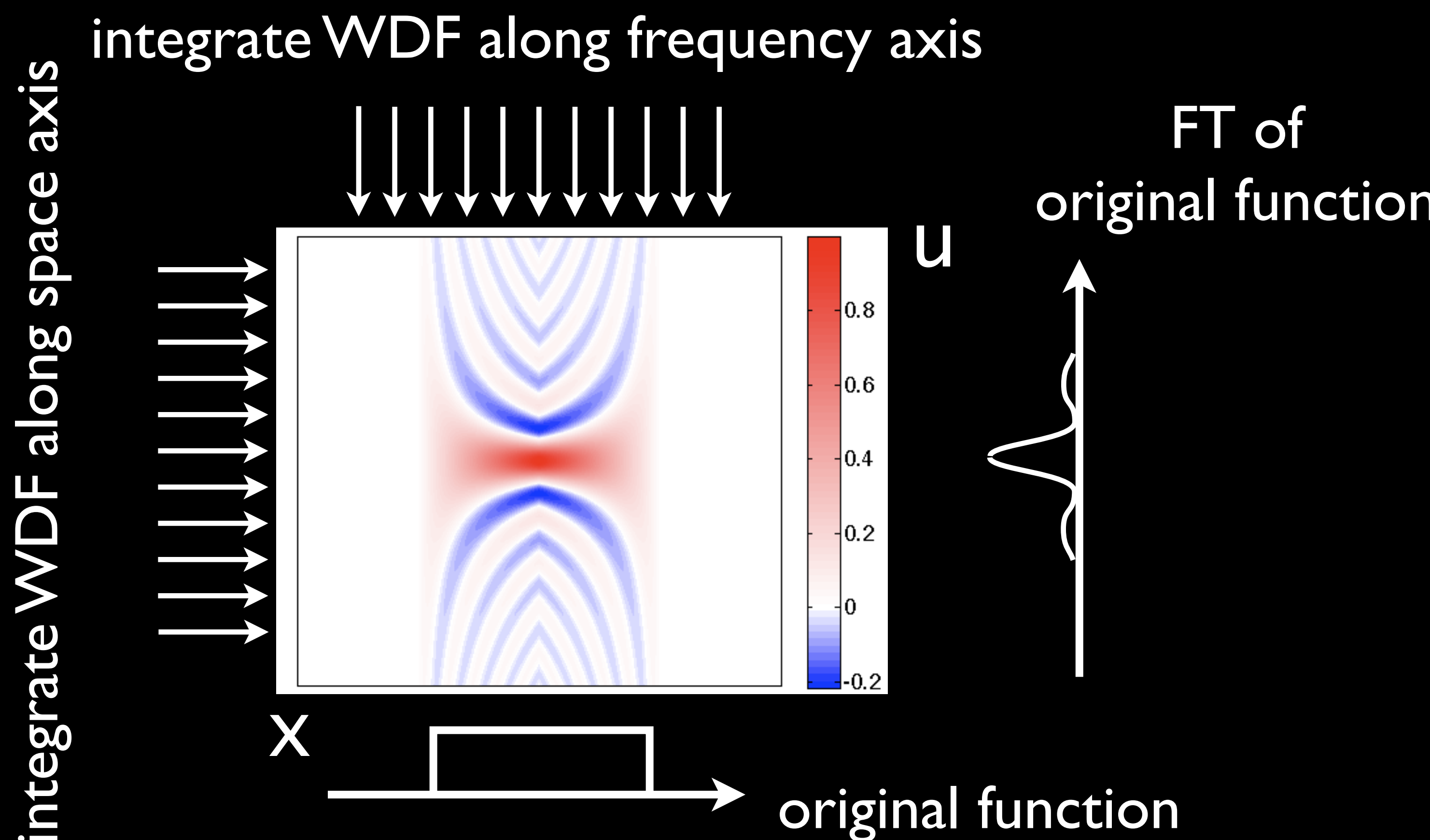


Wigner space
(x-u space)

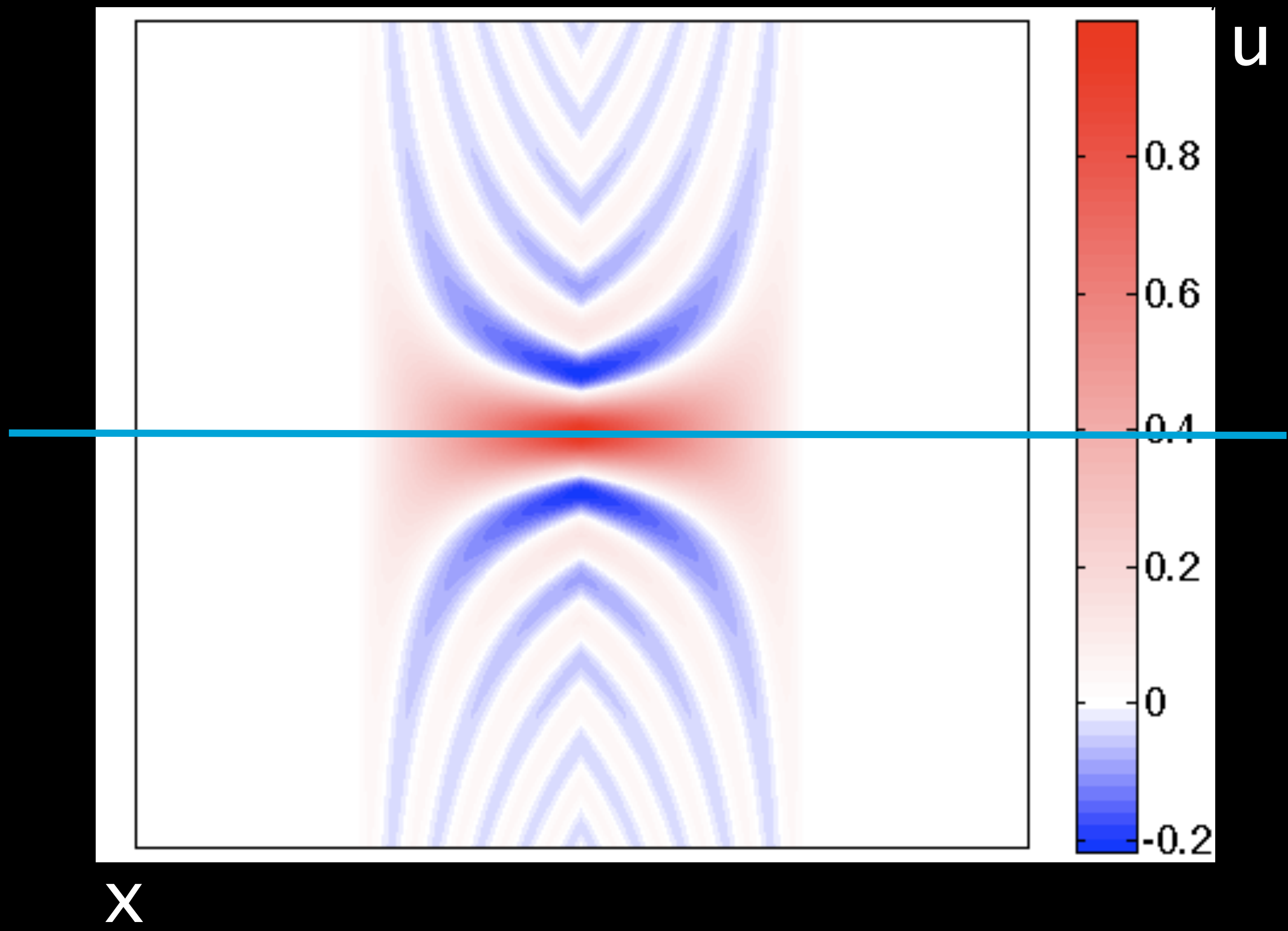
WDF ***shears/rotates*** upon propagation

boxcar ("rect") function



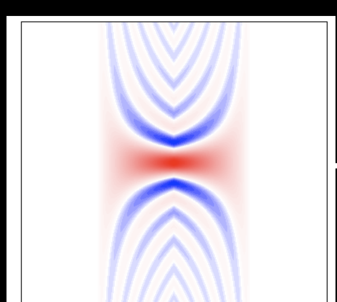


diffraction from a rectangular aperture



Wavefunction evolution and the WDF

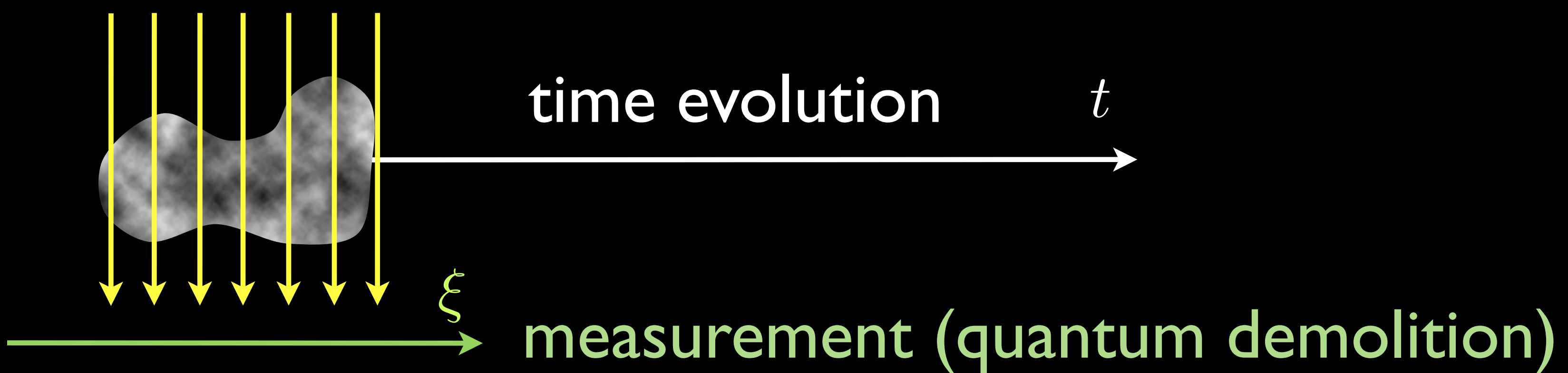
time evolution t

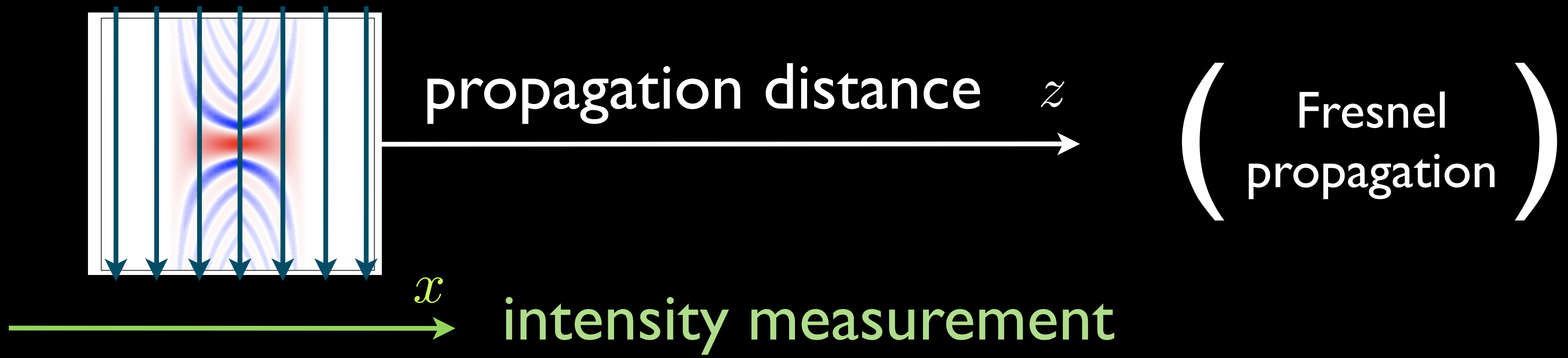


propagation distance z

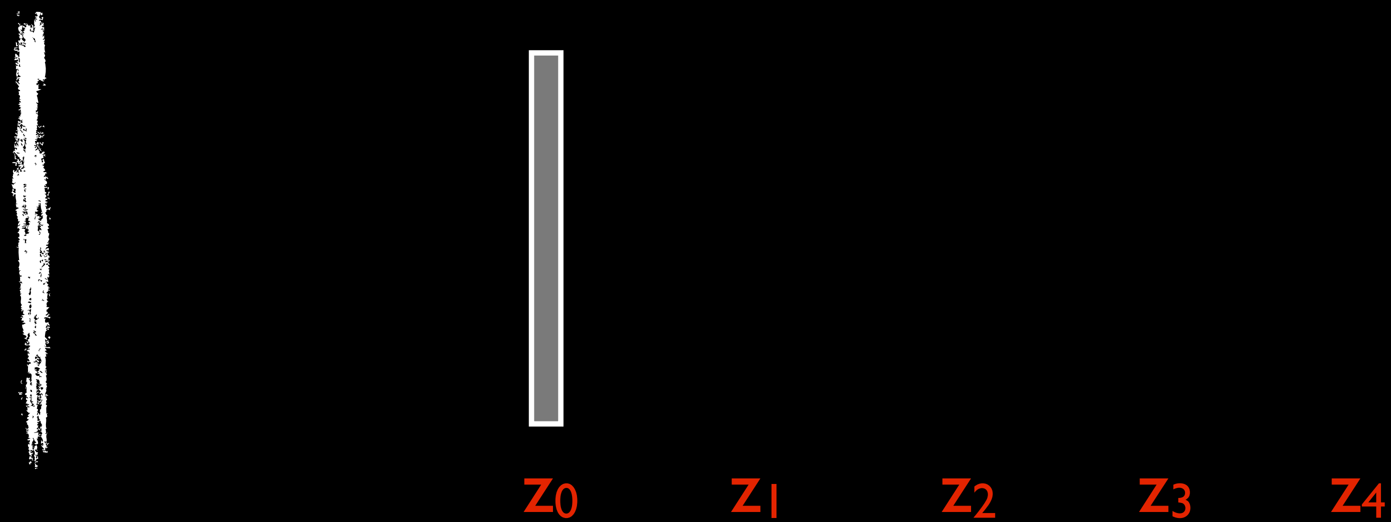
(Fresnel propagation)

Tomographic measurement from evolution/propagation





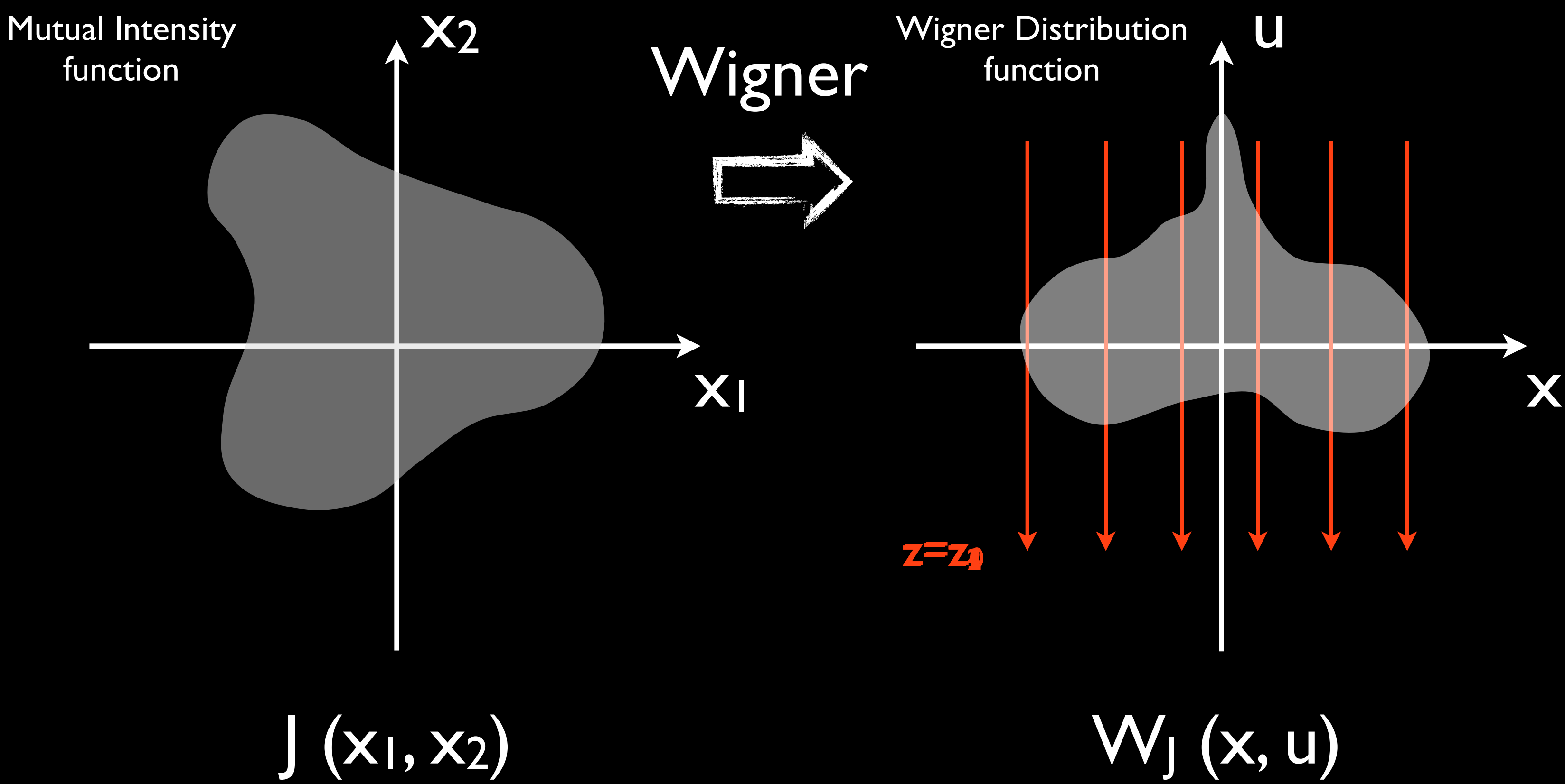
Phase-space tomography



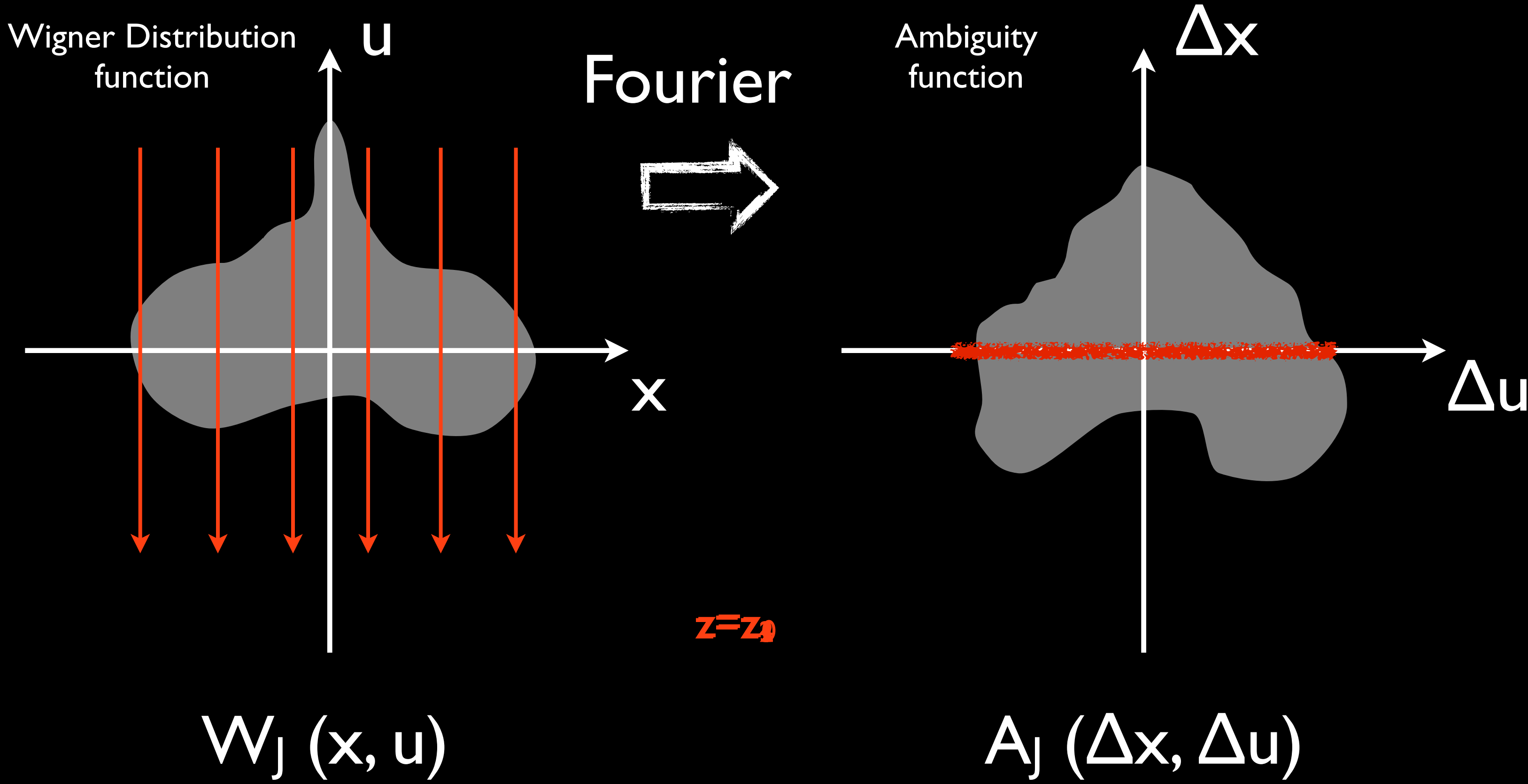
partially coherent field (unknown)

camera (intensity measurement)

Phase-space tomography

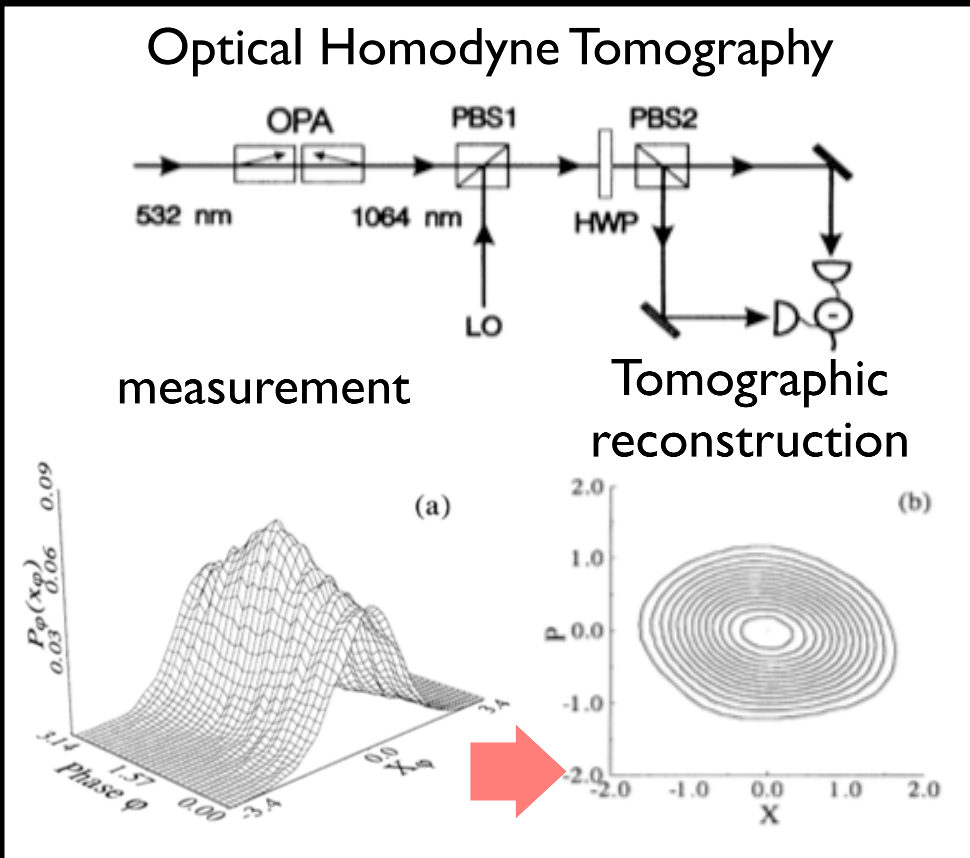


Phase-space tomography



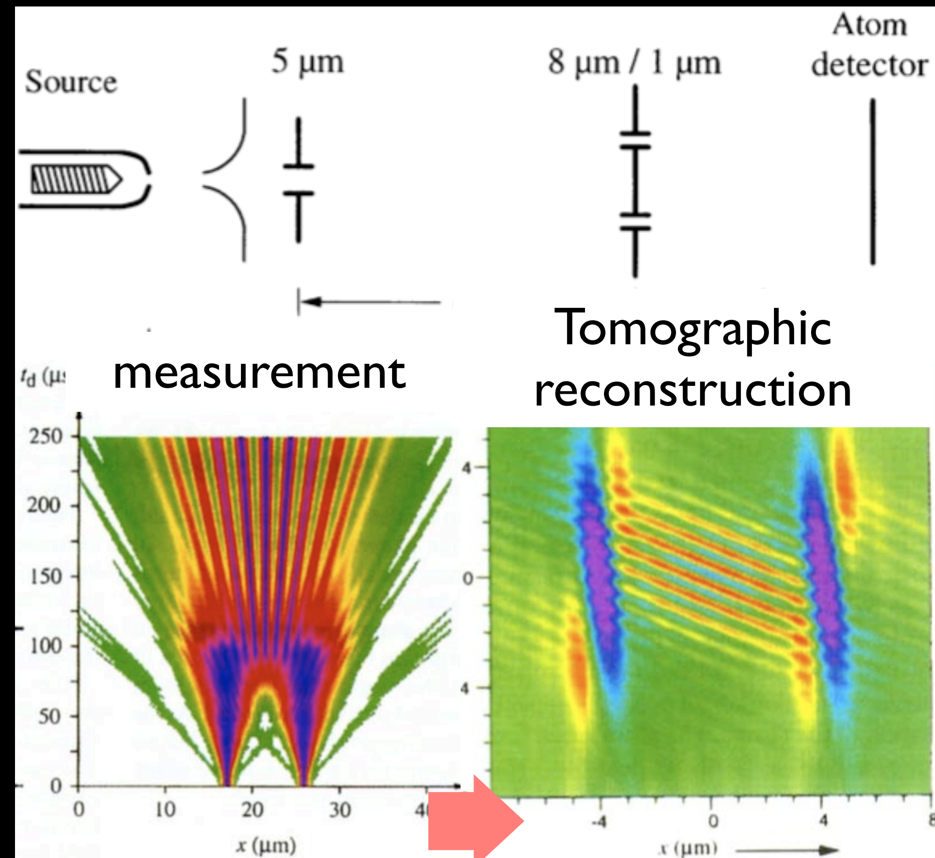
Quantum phase space tomography

Squeezed state recovery



D. Smithey, and et al, *Phys. Rev. Lett.* 1993

Matter wave interference



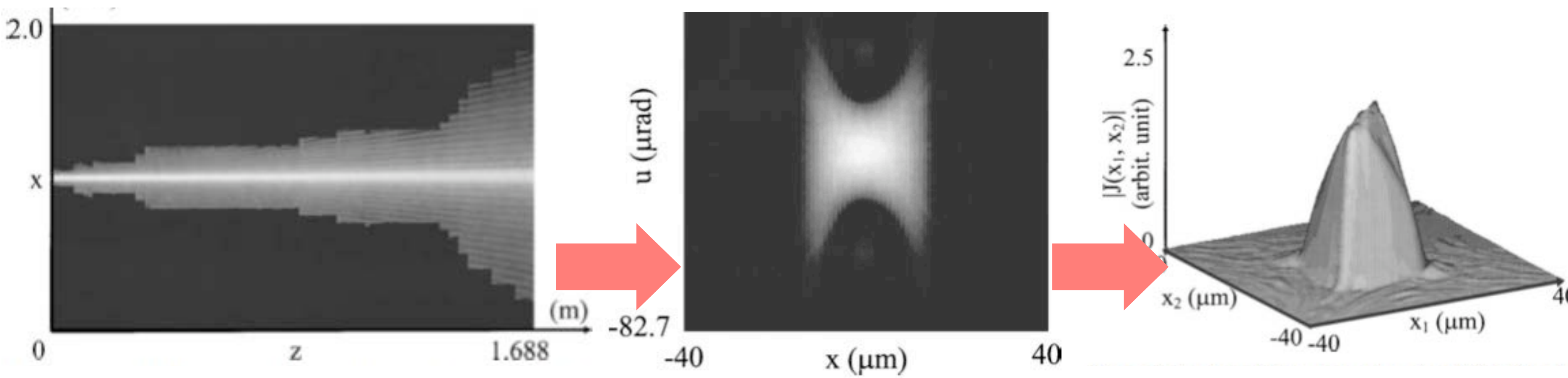
C. Kurtsiefer, and et al, *Nature*, 1997
J. Itatanl, and et al, *Nature*, 2000

Optical phase space tomography

- Non-interferometric technique

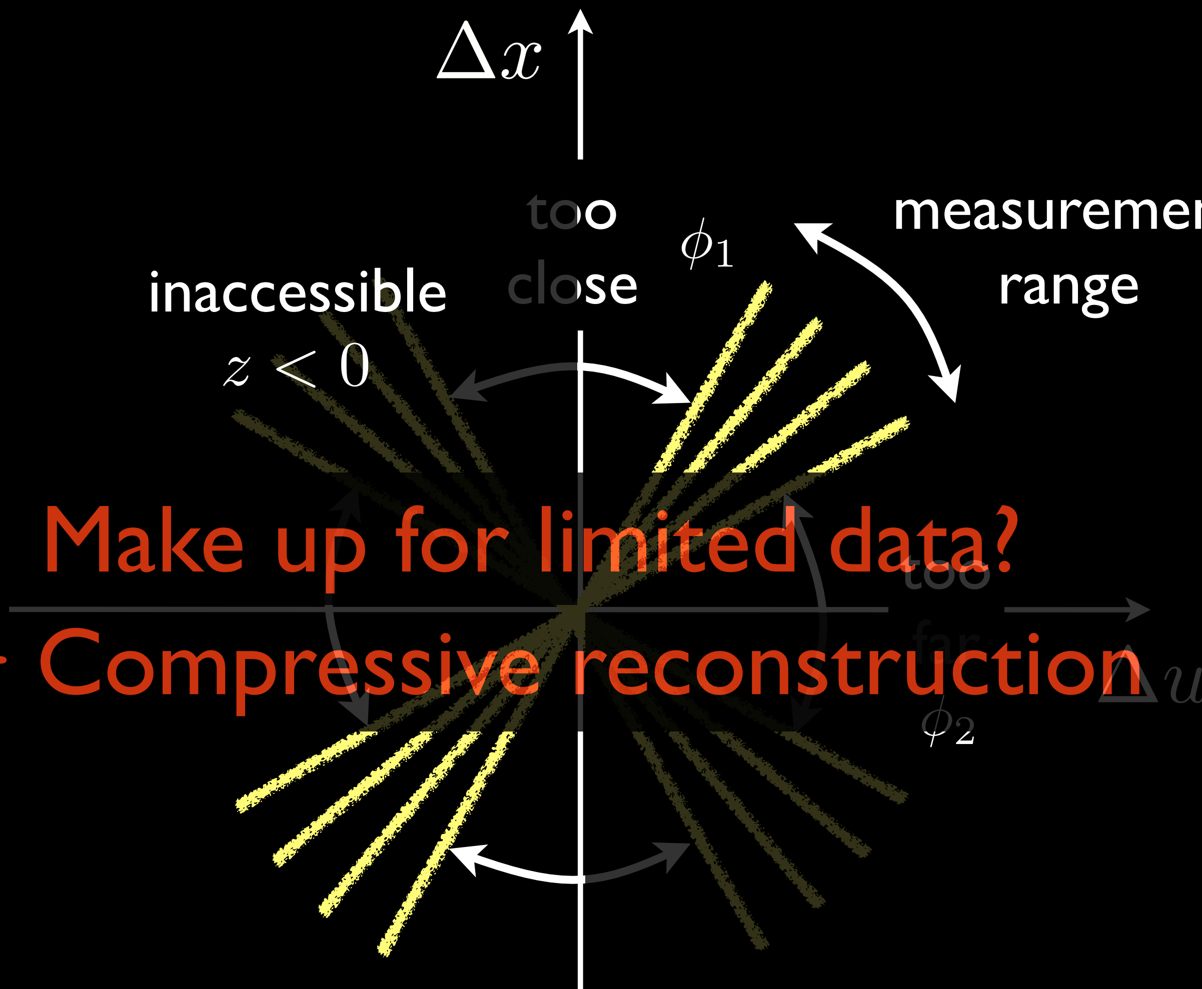
Spatial coherence measurements of a 1D soft x-ray beam

Axial intensity measurement Reconstructed WDF Reconstructed MI



C.Q.Tran, and et al, JOSA A 22, 1691-1700(2005)

The problem of limited data



Make up for limited data?

☛ Compressive reconstruction

- Assume intensity symmetric about *z=0*

Compressive reconstruction of phase-space data

- Sparsity claim: number of *coherent modes*
- Low-Rank Matrix Recovery
  - E. J. Candés and B. Recht, *Found. Comp. Math.* 9:717, 2009
  - L. Tian, J. Lee, S. B. Oh, and G. Barbastathis, *Opt. Express* 20:8296, 2012
- Factored Form Descent
  - Z. Zhang, S. Rehman, C. Zhi, and G. Barbastathis, *Opt. Express* 21:5759, 2013
- Sparse Kalman filtering
  - J. Zhong, L. Tian, R. A. Claus, J. Dauwels, and L. Waller, *FiO 2013* paper FW6A.9 (post-deadline, today)

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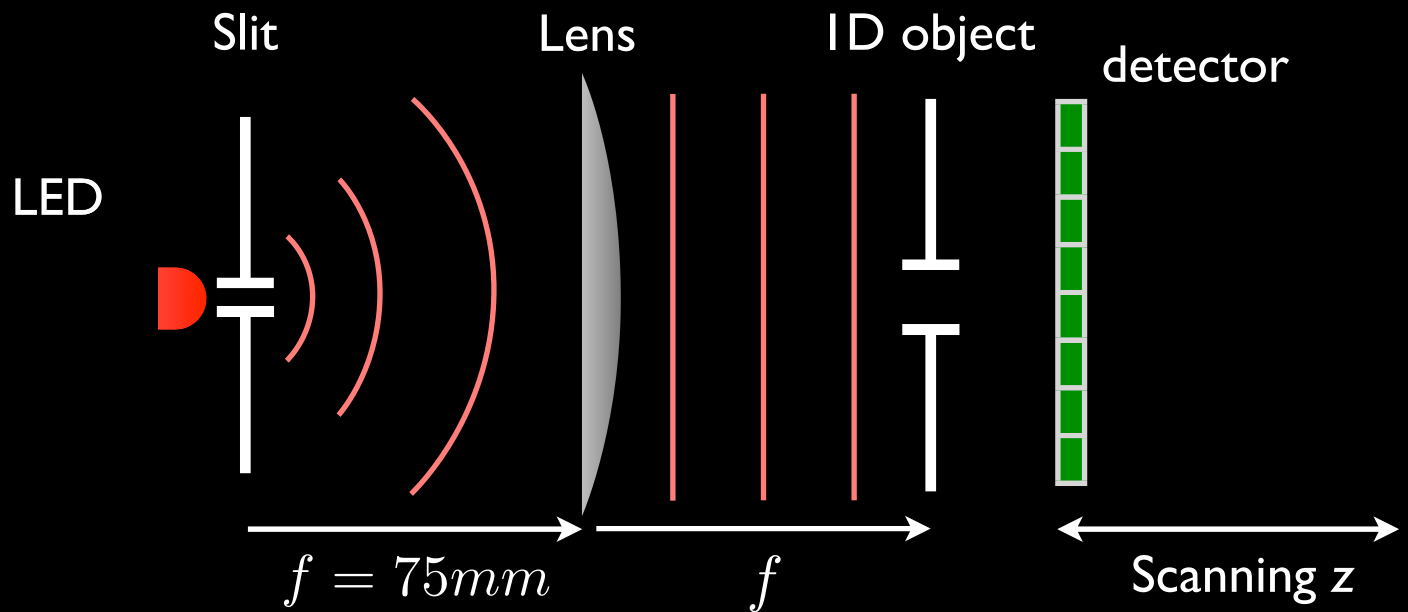
Low Rank Matrix Recovery (LRMR) of phase-space data

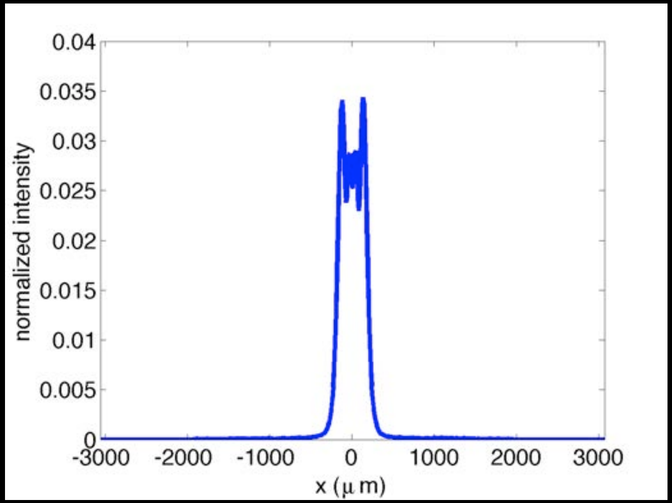
- Solution expressed as *few* coherent modes
- L0 minimization
  - sadly, intractable
  - however [Candes] the problem can be mapped onto an equivalent L1 minimization
- Semi-positive definiteness⇔mode coefficients≥0 additionally enforced as constraint

Exprerimental compressive phase-space tomography

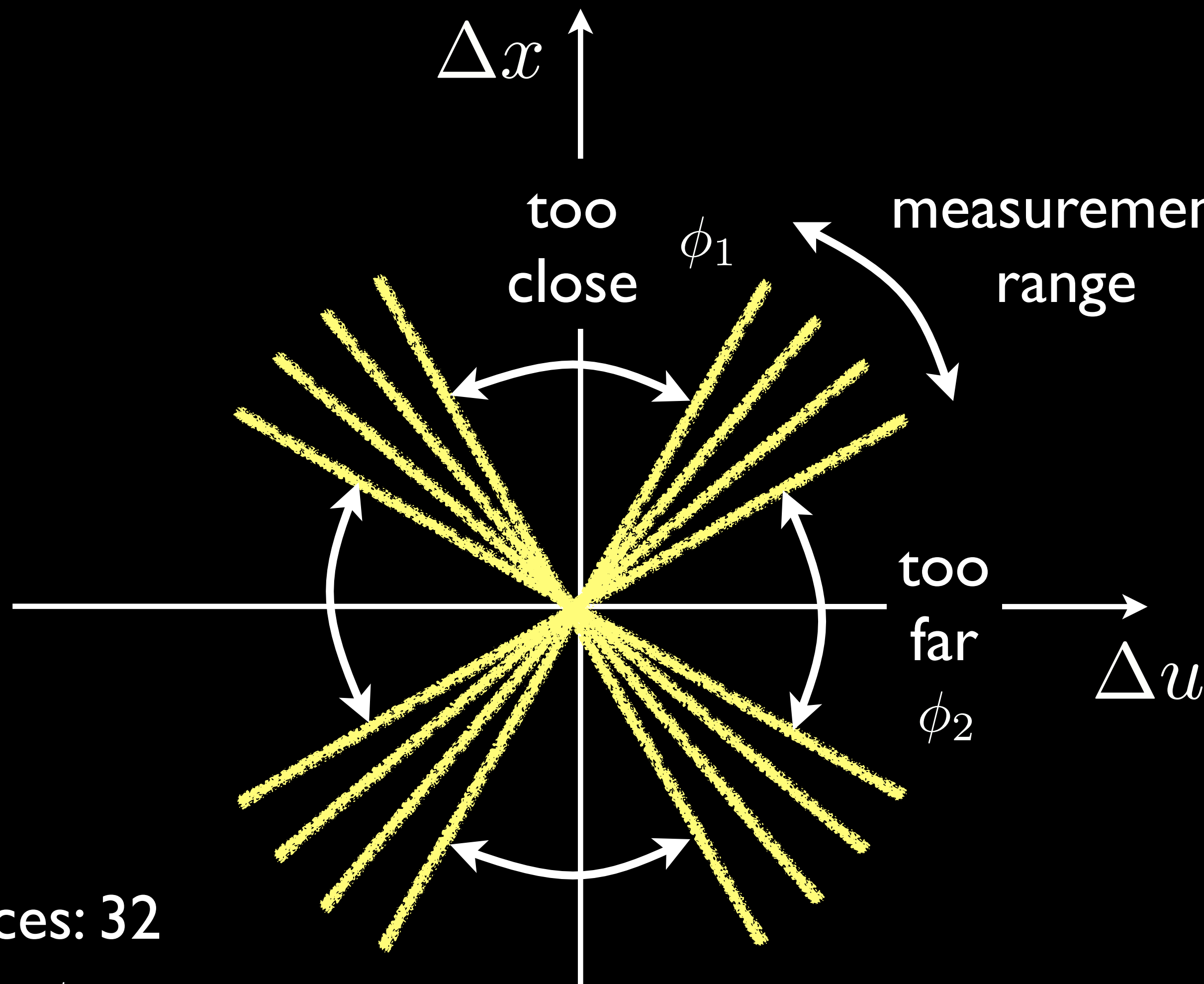


- Illumination central wavelength: 620nm; bandwith: 20nm
- Width of illumination slit: 300µm
  - Coherence length: 93µm
- Width of object slit: 400µm
- 32 measurements (axial positions)

Lei Tian et al, *Opt. Expr.* 20(8):8296, 2012

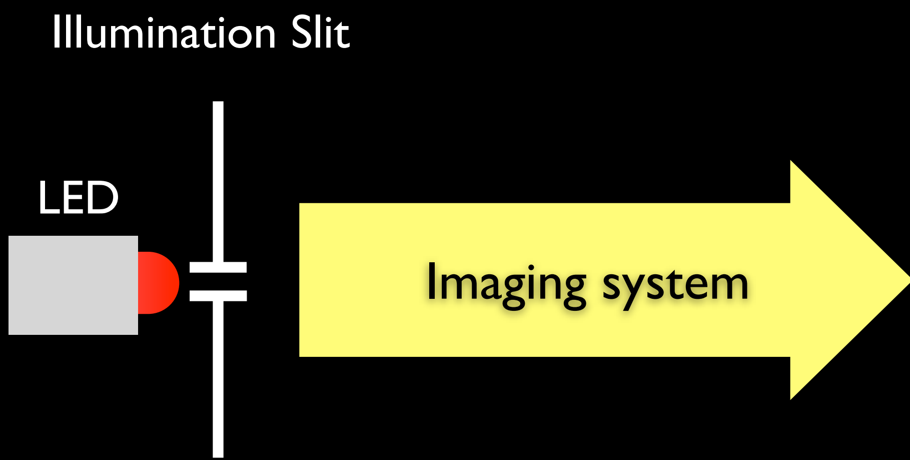


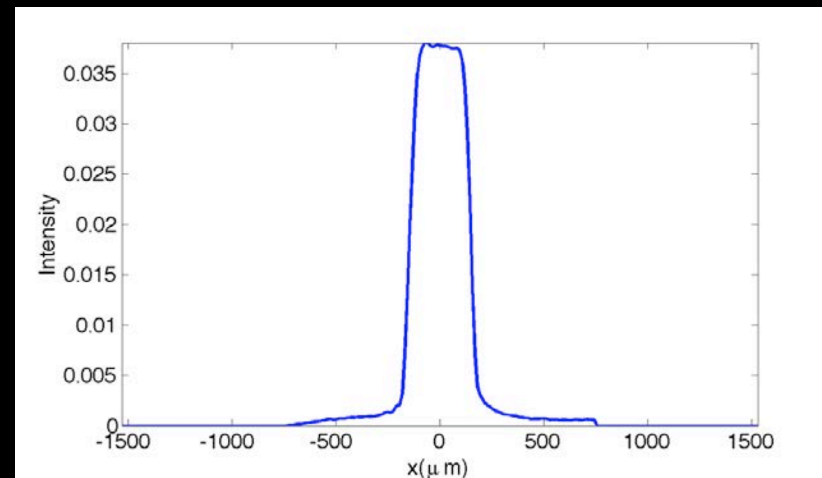
Limited data in our experiment



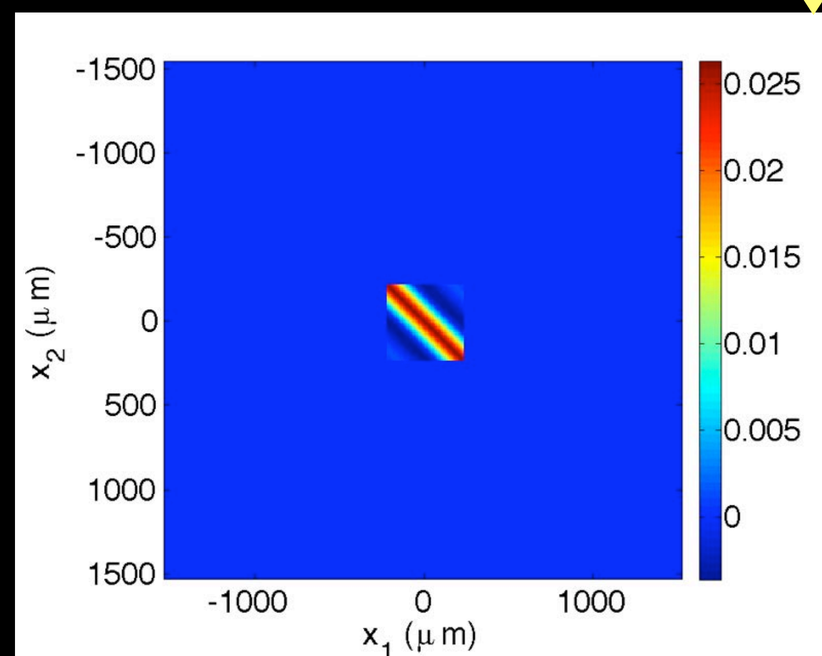
- Total # of slices: 32
- Missing angle ϕ_1: 38°
- Missing angle ϕ_2: 22°

Ground Truth





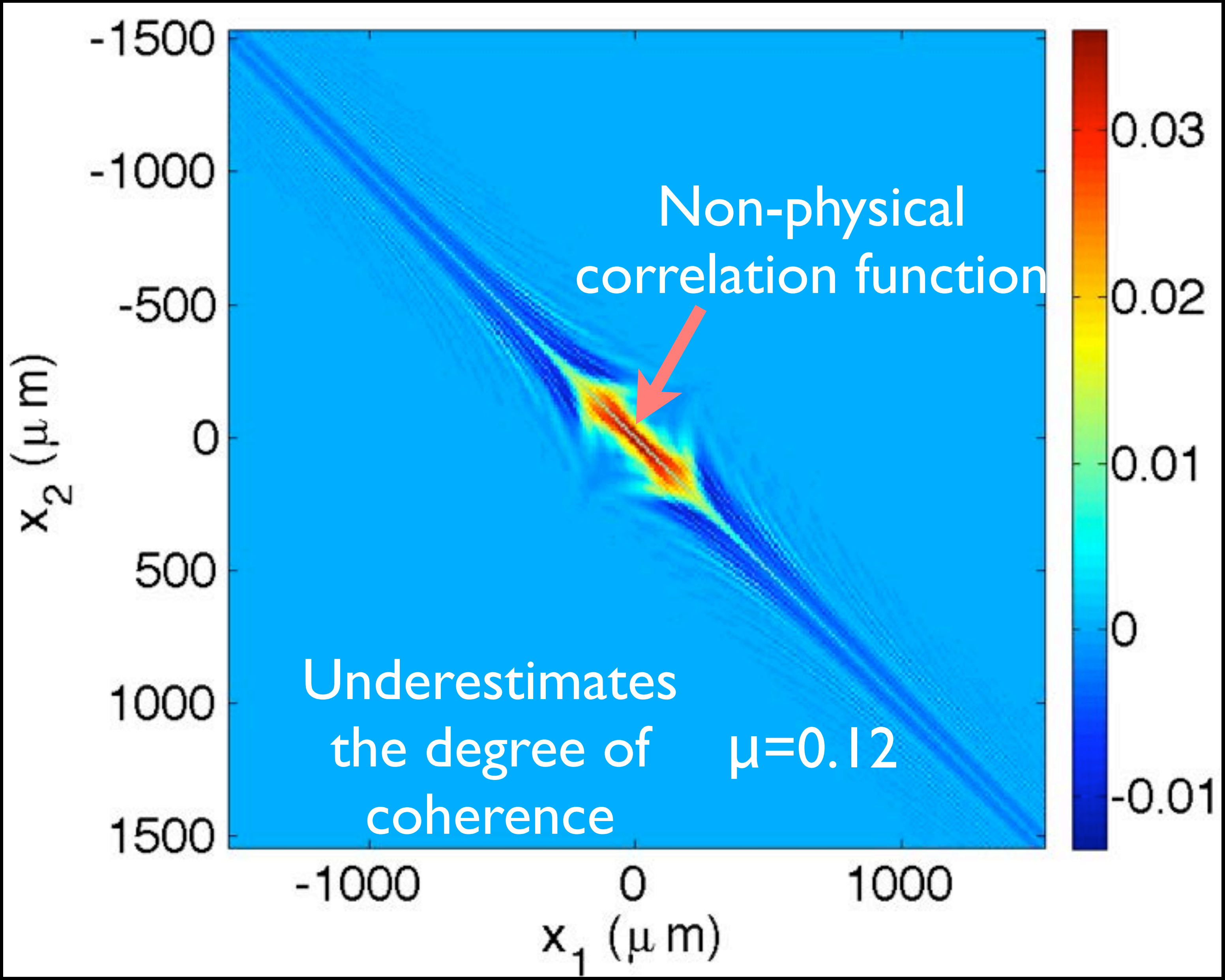
van Cittert-Zernike theorem



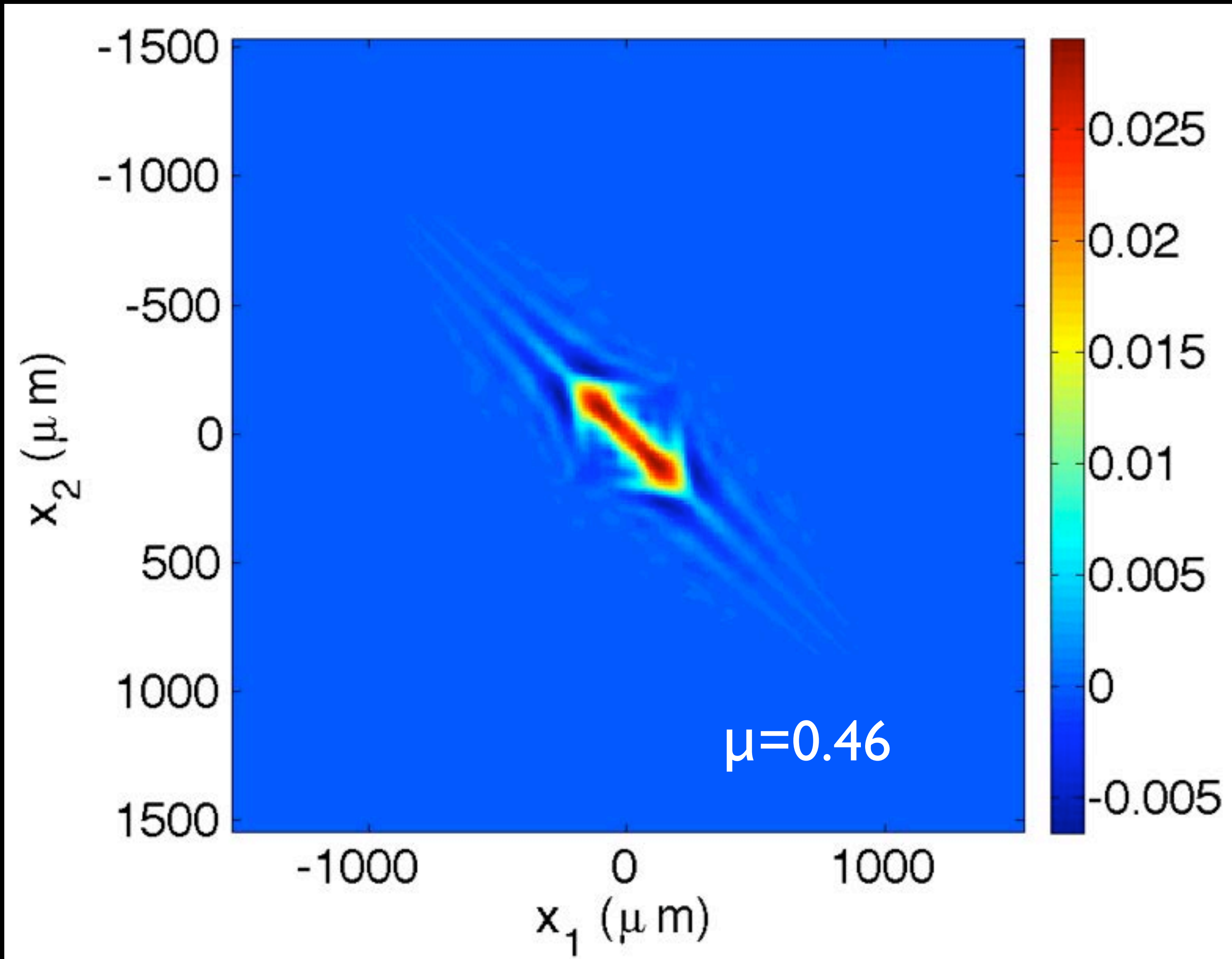
Global Degree of Coherence

μ=0.49

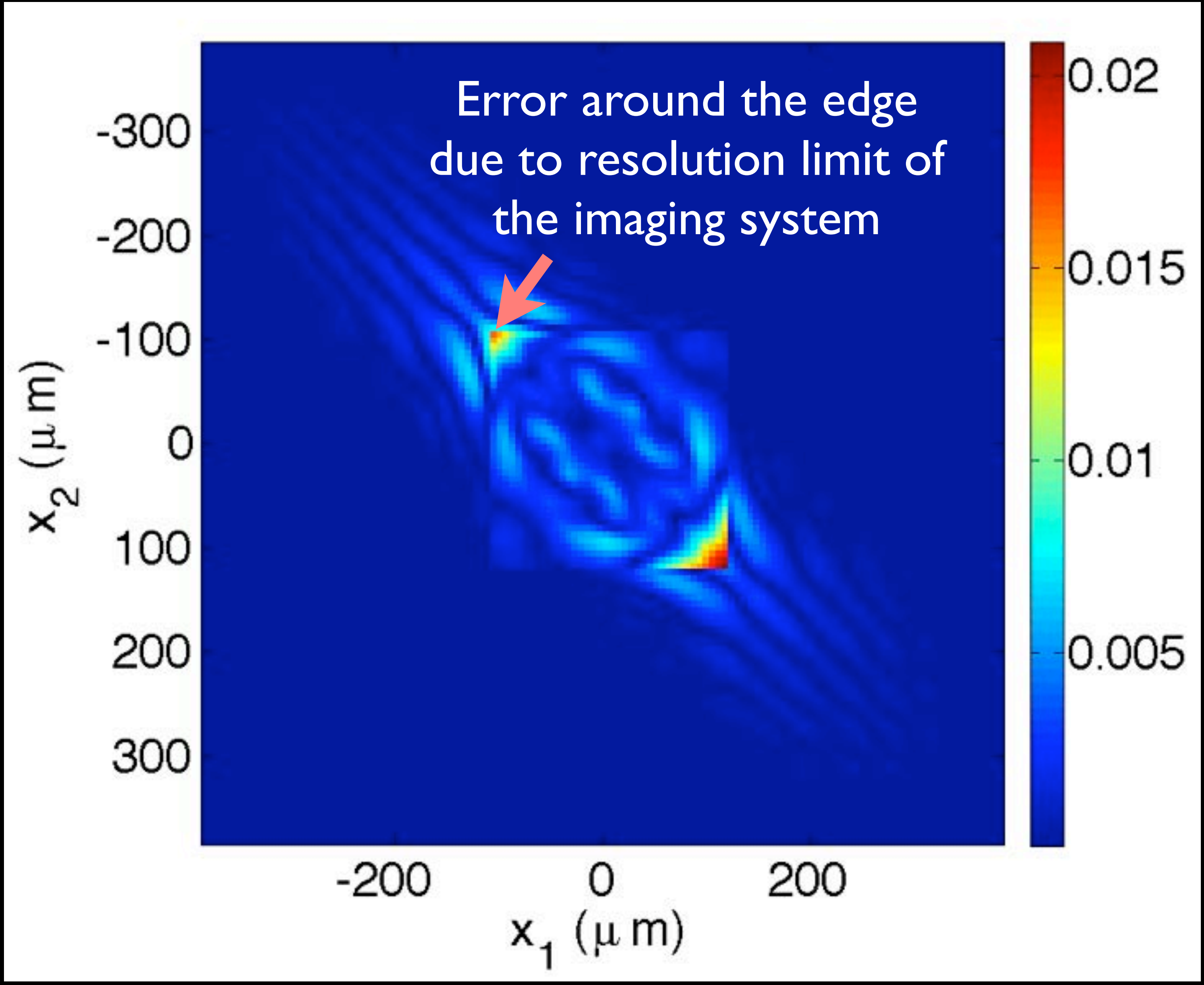
Filtered back-projection fails



Compressive reconstruction



Error in compressive reconstruction
(compared to Van Cittert-Zernike)



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Factored Form Descent

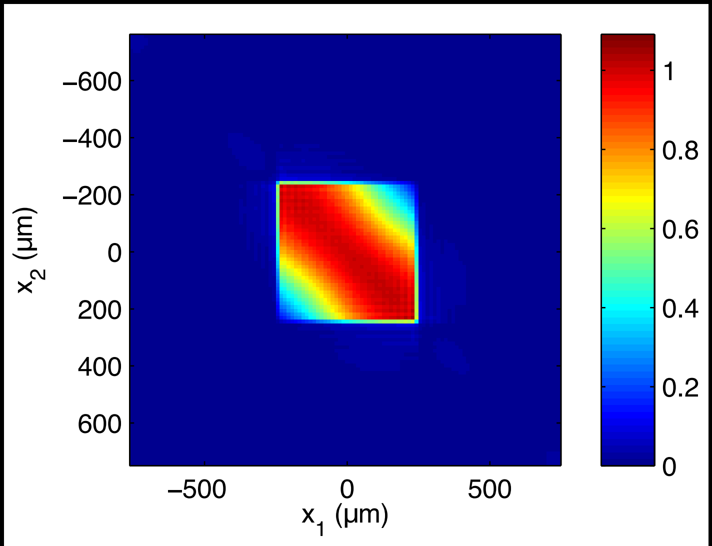
Wolf, *JOSA* 1982
Ozaktas et al., *JOSAA* 2002

- coherence mode decomposition: $J = UU^H$
- solve for U instead of J means semi-positive definiteness is automatically ensured
- quartic, need iterated descent algorithm
  - initialize with fully incoherent guess
  - find search direction, e.g. $\Delta U = ADA^H U$ (steepest descent, NL conjugate gradient)
  - perform (global) line search (solve for roots of cubic polynomial)
  - mutual intensity from singular value decomposition

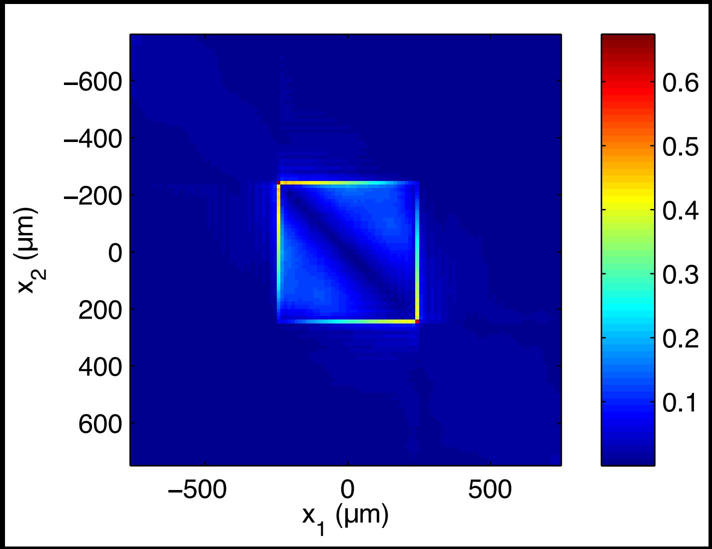
Z. Zhang, S. Rehman, C. Zhi, and G. Barbastathis, *Opt. Express* 21:5759, 2013

Experimental Test

51×401 intensity measurements
to compute 200×200 mutual intensity matrix



experimental



error compared to theory

(theory assumes perfect lenses, paraxial propagation, uniform LED, infinitesimal pixel size)

Phase from partially coherent fields

- Compressive reconstruction of the mutual intensity [Tian *Opt. Exp.* 20:8296]
- Factored Form Descent [Zhang *Opt. Exp.* 21:5756]
- Wigner distribution function recovery from lenslet arrays [Tian *Opt. Exp.* 21:10511]
- Optical Path Length (OPL) recovery with partially coherent illumination [Petruccelli *Opt. Exp.* 21:14430]

- **Thanks**
    - Laura Waller, David Brady, Colin Sheppard
    - Singapore's National Research Foundation
    - US Department of Homeland Security
    - Chevron Technology Company